Lossless for $S(s) \Rightarrow 1_2 = S^T(-s)S(s)$

or n for n-port

Power: $P(j\omega) = \mathrm{Re}\, v^{T*}(j\omega)\, i(j\omega)$

$$2P(j\omega) = i^{T*}(j\omega)\, v(j\omega) + v^{T*}(j\omega)\, i(j\omega)$$

$$\left.\begin{array}{l} v = v^i + v^r \\ i = v^i - v^r \end{array}\right\} \uparrow \quad = (v^{iT*} - v^{rT*})(v^i + v^r) + (v^{iT*} + v^{rT*})(v^i - v^r)$$

$$2P(j\omega) = v^{iT*}v^i - v^{rT*}v^r + \cancel{v^{iT*}v^r} - \cancel{v^{rT*}v^i}$$
$$+ v^{iT*}v^i - v^{rT*}v^r + \cancel{v^{iT*}(-v^r)} + \cancel{v^{rT*}v^i}$$

$$P(j\omega) = v^{iT*}v^i - (v^{iT*}S^{T*})(Sv^i) = v^{iT*}\left[1_2 - S^{T*}(j\omega)S(j\omega)\right]v^i$$

for passive want this > 0

v^i = incident voltage, $2v^i = v + i = v + z_0 i$ $\quad z_0$ normalized $= 1_2$ ($\Vert$ n)

v^r = reflected voltage, $2v^r = v - i = v - z_0 i$

$v^r = S v^i$; $S(s)$ is rational in s for a finite circuit

For passive $1_2 - S^{T*}(j\omega)S(j\omega) = 1_2 - \|S(j\omega)\|^2 \geq 0$ $\Uparrow$ for passive

also $S(s)$ has no poles in $\sigma \geq 0$

Lossless $\Leftrightarrow P(j\omega) = 0 \Rightarrow 1_2 = S^{T*}(j\omega)S(j\omega) = S^T(-j\omega)S(j\omega)$

$\uparrow$ rational real coefficients

$\Downarrow$

$1_2 = S^T(-s)S(s)$ by analytic continuation
(n)
(lossless)

Ex: $Av = Bi \Rightarrow A(v^i + v^r) = B(v^i - v^r)$

$$(A - B)v^i = -(A+B)v^r$$

$$\Rightarrow v^r = Sv^i \Rightarrow (B+A)^{-1}(B-A)v^i = v^r$$

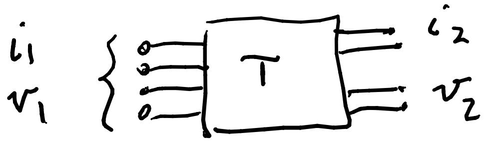
$$S = (B+A)^{-1}(B-A)$$

For transformer:

$$v_2 = T v_1 \qquad v_1^T i_1 + v_2^T i_2 = 0$$

$$i_1 = -T^T i_2 \qquad v_1^T i_1 + v_1^T T^T i_2 = 0$$

$$\Leftarrow$$



$$Av = Bi$$

$$\begin{bmatrix} -T & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 1 & T^T \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

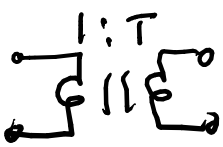
$Y \Rightarrow i = Yv$
$\quad = B^{-1}Av$

no B^{-1} for the transformer.

$$B+A = \begin{bmatrix} -T & 1 \\ 1 & T^T \end{bmatrix}; \text{ if } n=1 \quad -TT^T - 1$$

here $(B+A)^{-1}$ always exists if T is real

for $n=1$



then $(B+A)^{-1} = \dfrac{-1}{1+T^2}\begin{bmatrix} T & -1 \\ -1 & -T \end{bmatrix}$

$$(B-A) = \begin{bmatrix} T & -1 \\ 1 & T^T \end{bmatrix}$$

$$S = \frac{-1}{1+T^2}\begin{bmatrix} T & -1 \\ -1 & -T \end{bmatrix}\begin{bmatrix} T & -1 \\ 1 & T^T \end{bmatrix} = \frac{-1}{1+T^2}\begin{bmatrix} T^2-1 & -2T \\ -2T & 1-T^2 \end{bmatrix}$$

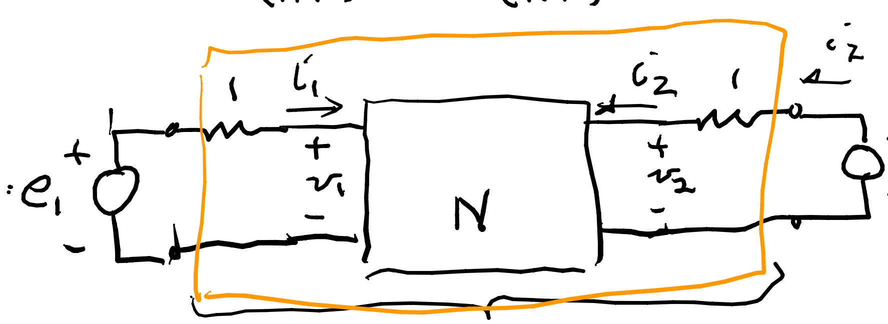
$S^{-1} = S^T$?

$$S^T = S$$

for S^{-1}, $\det S = \dfrac{(T^2-1)(1-T^2) - 4T^2}{(1+T^2)^2} = \dfrac{T^2 - T^4 - 1 + T^2 - 4T^2}{1+2T^2+T^4}$

$$= -\frac{(T^4+2T^2+1)}{1+2T^2+T^4} = -1$$

$$S^{-1} = \frac{1}{-1}\begin{bmatrix} \dfrac{1-T^2}{-(1+T^2)} & 2T/-(1+T^2) \\ \dfrac{+2T}{-(1+T^2)} & \dfrac{(T^2-1)}{-(1+T^2)} \end{bmatrix} = S^T \Rightarrow \text{lossless}$$



$$e_2 = v_2 + i_2 = 2v_2^i$$

a = augmented

N_a

$$i = Y_a e$$

$i = Y_a e = 2 Y_a v^i \qquad 2v^r = v - i$

$v = e - i = e - Y_a e$

$\quad = (\underline{1}_2 - Y_a) e$

$\Rightarrow \quad (\underline{1}_2 - Y_a) i = (\underline{1}_2 - Y_a) Y_a e = (Y_a - Y_a^2) e$

$Y_a \cdot v = Y_a (\underline{1}_2 - Y_a) e = (Y_a - Y_a^2) e$

$A v = B i$

$Y_a v = (\underline{1}_2 - Y_a) i \Rightarrow A = Y_a, \; B = \underline{1}_2 - Y_a$

$S = (B + A)^{-1} (B - A) = \underline{1}_2 \times (\underline{1}_2 - 2 Y_a) = \underline{1}_2 - 2 Y_a$

Y_a does not exist if we see $-1\,\Omega$ @ any port

essentially every passive circuit has an S