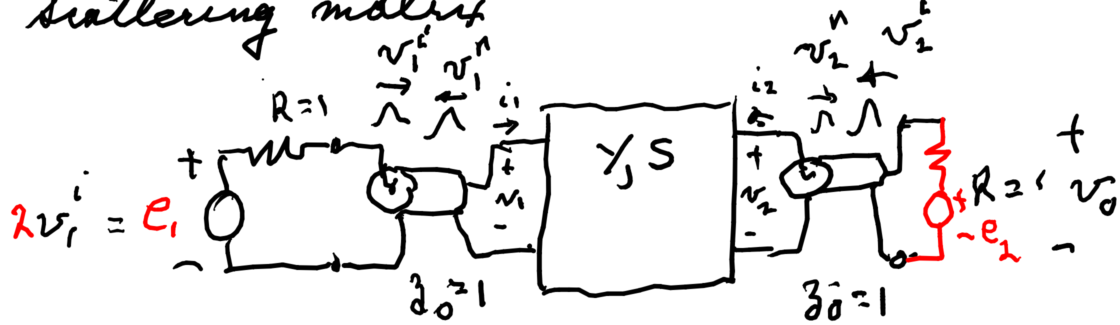
EE 610
10/29/13

Note Title 10/29/2013

p. 406 = adjoint scattering matrix



lines of length 0 to interpret

$$v^r = S v^i$$

S = scattering matrix

$$2v^i = v + z_0 i = v + i$$
$$2v^r = v - z_0 i = v - i$$

$$2v^i + 2v^r = 2v \Rightarrow v = v^i + v^r$$
$$2v^i - 2v^r = 2i \Rightarrow i = v^i - v^r$$

$v^i = \begin{bmatrix} v_1^i \\ v_2^i \end{bmatrix}$ $i \Rightarrow$ incident, $v = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$

$v^r = \begin{bmatrix} v_1^r \\ v_2^r \end{bmatrix}$ $r \Rightarrow$ reflected, $i = \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$

$$\rightarrow \tfrac{1}{2}(v - i) = S \cdot \tfrac{1}{2}(v + i)$$

$$\tfrac{1}{2} i + \tfrac{1}{2} S i = \tfrac{1}{2}(v - Sv)$$

$$(1_2 + S)\, i = (1_2 - S)\, v \Rightarrow i = Yv = (1_2 + S)^{-1}(1_2 - S)\, v$$

$$Y = (1_2 + S)^{-1}(1_2 - S)$$

$S_{11} = \left.\dfrac{v_1^r}{v_1^i}\right|_{v_2^i = 0}$; $v_2^i = \dfrac{v_2 + i_2}{2} \Rightarrow v_2 = -i_2$ if $v_2^i = 0$ $\Rightarrow e_2 = 0$

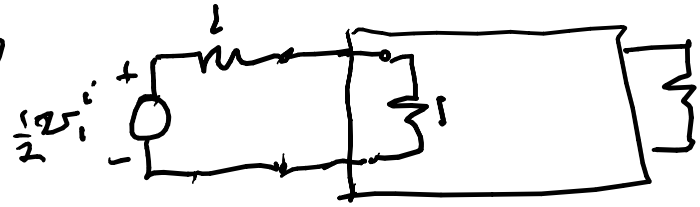
always $e_2 = v_2 + R i_2 = v_2 + i_2 = 2 v_2^i$

$$e_1 = v_1 + R i_1 = v + i_1 = 2 v_1^i$$

S_{11} = input S at port 1 when load in port 2 by $z_0 = 1$
= reflection coefficient @ port with $1\,\Omega$ on port 2

If $S_{11} = 0 \Rightarrow v_1^r = 0$ when $v_2^i = 0$
$= \tfrac{1}{2}(v_1 - i_1) \Rightarrow v_1 = i_1 \Rightarrow$ input @ port 1 = $1\,\Omega$ when load port 2 in $1\,\Omega$

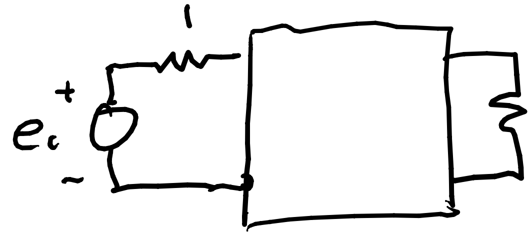
↓ if $S_{11}=0$



$$S_{21} = \frac{v_2^r}{v_1^i}\Big|_{v_2^i=0} \Rightarrow v_2 = -Ri_2 = -i_2 \;;\; v_2^r = \frac{1}{2}(v_2 - i_2) = v_2$$

$$v_1^i = \frac{1}{2}e_1$$

$$= \frac{v_2}{\frac{1}{2}e_1} = 2\frac{v_2}{e_1} = S_{21}$$



$$v_2 = \frac{1}{2}S_{21}\cdot e_1$$

=> called transmission coefficient

Note: $(1_2+S)^{-1}(1_2-S) = (1_2-S)(1_2+S)^{-1}$

to see ×(1_2+S) on left & right; $(1_2-S)(1_2+S)$, $(1_2+S)(1_2-S)$

$$1_2 - S^2 \Longleftrightarrow 1_2 - S^2$$

are =

For a 1-port $1_2 \to 1$

Note $Y = (1_2+S)^{-1}(1_2-S) \Rightarrow (1_2+S)Y = (1_2-S)$

$$Y + SY = 1_2 - S$$

$$Y - 1_2 = (-S - SY) = S(-1-Y)$$

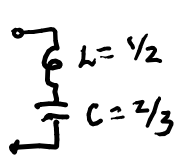
$$S = (1_2 - Y)(1_2 + Y)^{-1}$$

Ex:

$1_2 \to 1$, $y(s) = sC$ pole @ ∞

$$S(s) = \frac{(1-y(s))}{(1+y(s))} = \frac{1-sC}{1+sC}$$ no pole @ ∞ $(= -c/c)$

Ex $y(s) = \frac{2s}{s^2+3}$



$$S(s) = \frac{1-\frac{2s}{s^2+3}}{1+\frac{2s}{s^2+3}} = \frac{s^2+3-2s}{\underbrace{s^2+3+2s}_{\text{Hurwitz}}}$$

$$s_{1,2} = -\frac{2}{2} \pm \frac{1}{2}\sqrt{4-4\times 3} = -1 \pm j\sqrt{2} \text{ in LHP}$$

For passive circuits S is bounded-real & if rational called BR

conditions

1) $S(s)$ is real for real $s = \sigma > 0$
2) $S(s)$ is analytic in $\sigma \geq 0$
3) $1_2 - S^{T*}(s) \cdot S(s)$ is positive semi-definite in $\sigma \geq 0$ (bounded norm by 1)

For lossless: $Y(s) + Y^T(-s) = 0_2$

$$\underbrace{(1_2 + S(s))^{-1}(1_2 - S(s))}_{= Y(s)} + \underbrace{(1_2 - S^T(-s))(1_2 + S^T(-s))^{-1}}_{= Y^T(-s)}$$

$\times (1_2 + S(s))$ on left & $\times (1_2 + S^T(-s))$ on right

$$(1_2 - S(s))(1_2 + S^T(-s)) + (1_2 + S(s))(1_2 - S^T(-s)) = 0_2$$

$$1_2 - \cancel{S(s)} + \cancel{S^T(-s)} - S(s)S^T(-s) + 1_2 - \cancel{S^T(-s)} + \cancel{S(s)} - S(s)S^T(-s) = 0_2$$

$$2 \cdot 1_2 - 2 S(s) S^T(-s) = 0_2 \Rightarrow S(s) \cdot S^T(-s) = 1_2$$

$$\Rightarrow S(s)^{-1} = S^T(-s) \Leftrightarrow \text{lossless condition}$$