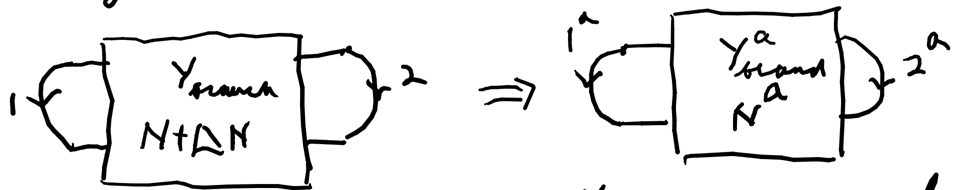
EE610
10/24/13

Note Title 10/24/2013

$$S_x^{T(x)} = \frac{dT(x)/dx}{T(x)/x} = \frac{x}{T(x)} \cdot \frac{dT(x)}{dx}$$

use adjoint circuit

adjoint



these have the same graph

$$i_b^T v_b = 0 \qquad i_b^{aT} v_b^a = 0$$

also $i_b^T v_b^a = 0$ & $i_b^{aT} v_b = 0$

$$\begin{bmatrix} i_1 \\ i_2 \\ \underline{i}_b \end{bmatrix} = i_b, \quad \begin{bmatrix} v_1 \\ v_2 \\ \underline{v}_b \end{bmatrix} = v_b$$

$$i_b^T v_b^a - i_b^{aT} v_b = 0$$

$$i_1 v_1^a + i_2 v_2^a + \underline{i}_b^T \underline{v}_b^a - i_1^a v_1 - i_2^a v_2 - \underline{i}_b^{aT} \underline{v}_b = 0$$

assume $T(x) = v_2/v_1{}^{(x)}$ keeps v_1 fixed but v_2 will change when change circuit & keeps N^a fixed

$$\underline{i}_b = Y \underline{v}_b, \quad \underline{i}_b^a = Y^a \underline{v}_b^a$$

$$\Delta \underline{i}_b = \Delta Y \cdot \underline{v}_b + Y \cdot \Delta \underline{v}_b, \quad \Delta \underline{i}_b^a = 0, \quad \Delta Y^a = 0, \quad \Delta \underline{v}_b^a = 0$$

after Δv_2, as $\Delta T(x) = \Delta v_2 / v_1$

apply changes to

$$\Delta i_1 \cdot v_1^a + i_1 \overset{=0}{\Delta v_1^a} + \Delta i_2 \cdot v_2^a + i_2 \cdot \overset{=0}{\Delta v_2^a} + \Delta \underline{i}_b^T \cdot \underline{v}_b^a + \underline{i}_b^T \cdot \overset{=0}{\Delta \underline{v}_b^a}$$

$$- \overset{=0}{\Delta i_1^a} v_1 - i_1^a \overset{=0}{\Delta v_1} - \overset{=0}{\Delta i_2^a} v_2 - i_2^a \Delta v_2 - \overset{=0}{\Delta \underline{i}_b^{aT}} \cdot \underline{v}_b - \underline{i}_b^{aT} \Delta \underline{v}_b = 0$$

$$\left(\Delta i_1 v_1^a + \Delta i_2 \cdot v_2^a + \underbrace{\Delta \underline{i}_b^T \cdot \underline{v}_b^a}\right) - i_2^a \Delta v_2 - \underbrace{\underline{i}_b^{aT} \Delta \underline{v}_b}$$

$$\Delta \underline{i}_b = \Delta Y \cdot \underline{v}_b + Y \cdot \Delta \underline{v}_b \qquad \underline{i}_b^a = Y^a \underline{v}_b$$

$$\underbrace{\left(\Delta i_1 v_1^a + \Delta i_2 v_2^a - i_2^a \underline{\Delta v_2}\right)}_{\text{external changes}} + \left[\underline{v}_b^T \Delta Y^T \underline{v}_b^a + \Delta \underline{v}_b^T Y^T \underline{v}_b^a\right] - \left[\underline{v}_b^{aT} Y^{aT} \Delta \underline{v}_b\right] = 0$$

interested in

$$\underline{v}_b^{aT} Y \Delta \underline{v}_b - \underline{v}_b^{aT} Y^{aT} \Delta \underline{v}_b = \underline{v}_b^{aT}\left[Y - Y^{aT}\right]\Delta \underline{v}_b$$

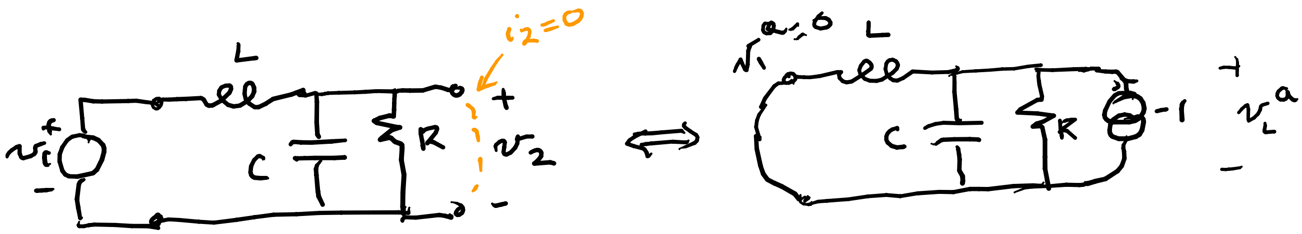
set this to zero choose N^a by $Y = Y^{aT} \Rightarrow Y^a = Y^T$

choose $v_1^a = 0 \Rightarrow$ a short, $i_2^a = -1$ (a current source)

$\Delta i_2 = 0$ (an open on port 2 of N)

$$\Delta v_2 = \underline{v}_b^T \Delta Y^T \underline{v}_b^a = \underline{v}_b^{aT} \Delta Y \underline{v}_b$$

$$\therefore \quad \frac{\Delta v_2}{\Delta x} = \underline{v}_b^{aT} \frac{\Delta Y}{\Delta x} \cdot \underline{v}_b$$



$$Y_{2\text{-port}} = \begin{bmatrix} \frac{1}{sL} & -\frac{1}{sL} \\ -\frac{1}{sL} & \frac{1}{sL} + Cs + G \end{bmatrix}; \quad Y^a = Y$$

$G = 1/R$

$$\underline{v}_b^T = [v_L \; v_C \; v_R], \qquad \underline{i}_b = \begin{bmatrix} \frac{1}{Ls} & 0 & 0 \\ 0 & Cs & 0 \\ 0 & 0 & G \end{bmatrix} \underline{v}_b; \quad Y = \begin{bmatrix} \frac{1}{Ls} & 0 & 0 \\ 0 & Cs & 0 \\ 0 & 0 & G \end{bmatrix}$$

$$\underline{v}_b^{aT} = [v_L^a \; v_C^a \; v_R^a]$$

if $S_L^{v_2/v_1}$, $x = L$ then

$$\frac{\Delta Y}{\Delta L} = \begin{bmatrix} -\frac{1}{L^2} \cdot \frac{\Delta L}{s} \cdot \frac{1}{\Delta L} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} -\frac{1}{L^2 s} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\therefore \quad \frac{\Delta v_2}{\Delta L} = -\frac{1}{L^2 s} \cdot v_L \cdot v_L^a \Rightarrow S_L^{v_2/v_1} = \frac{L}{\frac{v_2}{v_1}} \times \frac{-1}{L^2 s} \cdot v_L v_L^a$$

$$v_L^a = \frac{1}{\frac{1}{sL} + sC + G} \cdot (-1) \qquad v_L = \frac{sL}{sL + \frac{1}{Cs + G}} \times v_1 = 1$$

$$\frac{v_2}{v_1} = \frac{\frac{1}{Cs+G}}{sL + \frac{1}{Cs+G}} = \frac{1}{s^2LC + GLs + 1} \;; \quad \frac{d\, v_2/v_1}{dL} = \frac{-1\,(Cs^2+Gs)}{(s^2LC+GLs+1)^2}$$

by direct differentiation

$$\frac{\Delta v_2}{\Delta L} = -\frac{1}{L^2 s}\left(\frac{sL}{sL + \frac{1}{Cs+G}}\right) \cdot \frac{1}{\frac{1}{sL} + sC + G}$$

from N&N[a]

$$= \frac{-s\,(Cs+G)}{sL(Cs+G)+1} \times \frac{1}{1 + s^2LC + GLs} = \frac{-s\,(Cs+G)}{(CLs^2+GLs+1)^2}$$

$$S_L^{T(s)} = \frac{-\frac{1}{\cancel{Ls}}}{\frac{1}{\cancel{s^2LC+GLs+1}}} \cdot \frac{-\cancel{sL}}{\cancel{s^2LC+LGs+1}} \cdot \frac{sL(Cs+G)}{(s^2LC+LGs+1)} = \frac{sL(Cs+G)}{LCs^2+LGs+1}$$

varies with $s = \sigma + j\omega$