EE 610
10/17/13

Note Title 10/17/2013

$P(s) = 1 + (-1)^n s^{2n}$; desire zeroes of this

$$s^{2n} = \frac{-1}{(-1)^n} = \begin{cases} -1 & \text{if } n \text{ even} \\ +1 & \text{if } n \text{ odd} \end{cases}$$

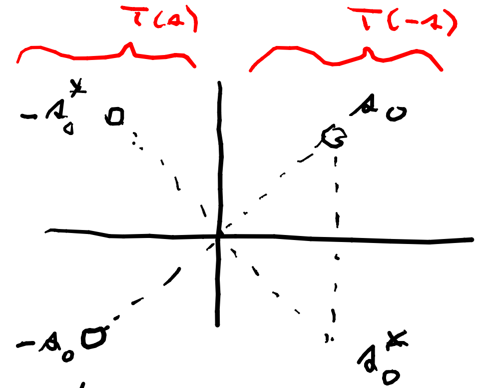
n even: $\quad -1 = e^{j(\pi + 2k\pi)}$; $k = 0, +1, \dots, 2n-1$

$$s_{roots} = e^{j(1+2k)\pi/2n}$$

$$P(s) = \prod_{k=0}^{2n-1} (s - s_k)$$

$$= P(-s)$$

$$= \frac{1}{T(s)T(-s)}$$



$n=2,\ k=0$

if $n=2$, s^{2n} gives 4 roots

$$s_0 = e^{j\pi/4} = \cos\pi/4 + j\sin\pi/4$$
$$= \frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}$$

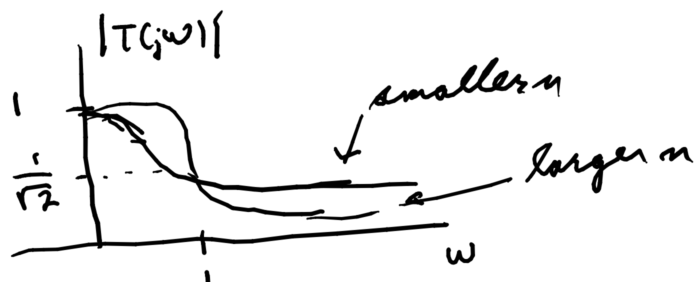
$n=2$

$$\frac{1}{T(s)T(-s)} = (s-s_0)(s+s_0)(s-s_0^*)(s+s_0^*)$$
$$= \left(s - \frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}\right)\left(s + \frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}\right)\left(s - \frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}\right)\left(s + \frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}\right)$$

$$T(s) = \frac{1}{\left(s + \frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}\right)\left(s + \frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}\right)} = \frac{1}{s^2 + \frac{2}{\sqrt{2}}s + 1}$$

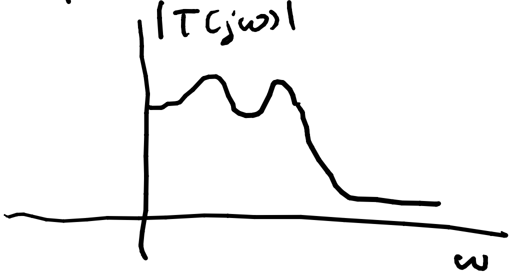
$$= \frac{1}{s^2 + \sqrt{2}s + 1} \Rightarrow \text{maximally flat low pass } T(s) \text{ @ } s=0$$

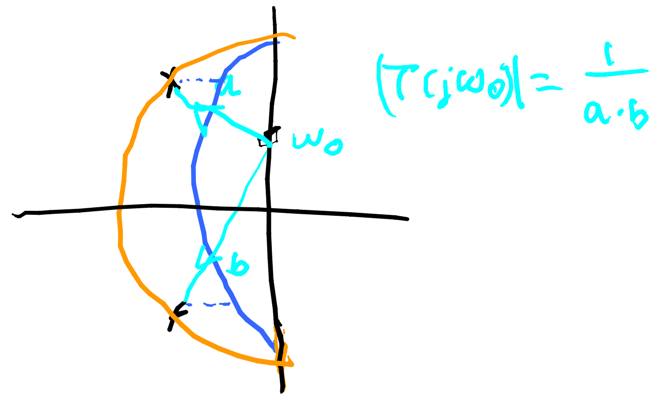
if n is odd there is a real root in $\frac{1}{T(s)}$ & that is on the unit circle so it is at $s = -1$



$\frac{1}{T(s)}$ = Butterworth polynomial

If move unit circle to an ellipse we get equal ripple





can make high pass by $s \to 1/s$

& band pass by $s = \frac{\omega_0}{p} + \frac{p}{\omega_0}$

Synthesis: $$T(s) = \frac{c_{n-1}s^{n-1} + \dots c_0}{s^n + a_{n-1}s^{n-1} + \dots + a_1 s + a_0} = \frac{v_o}{v_i} = \frac{y}{u}(s)$$

$$\begin{bmatrix} \dot{x}_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & 0 \dots & 0 \\ \vdots & & & & 0 \\ 0 & & & \ddots & 1 \\ -a_0 & -a_1 & & \dots & -a_{n-1} \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} + \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} u \quad , \quad \dot{} = \frac{d}{dt} = s$$

$$y = [c_0, \dots, c_{n-1}] \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

$$\Rightarrow \dot{x}_1 = s x_1 = x_2 \quad , \quad \dot{x}_2 = s(s x_1) = s^2 x_1$$

$$\dots \quad s^n x_1 = \dot{x}_n = -a_0 x_1 - a_1 s x_1 \dots - a_{n-1} s^{n-1} x_1 + u$$

$$(s^n + a_{n-1}s^{n-1} + \dots a_0) x_1 = u$$

$$y = [c_0, c_1, \dots, c_{n-1}] \begin{bmatrix} x_1 \\ s x_1 \\ \vdots \\ s^{n-1} x_1 \end{bmatrix} = (c_0 + c_1 s + \dots c_{n-1} s^{n-1}) x_1$$

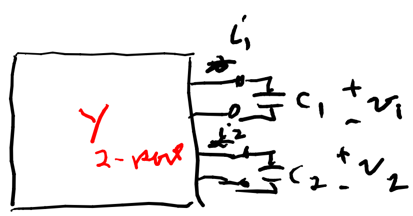
$$y = \frac{(c_0 + \dots c_{n-1}s^{n-1})}{s^n + \dots + a_0} \cdot u \Rightarrow \frac{y}{u} = \frac{c_{n-1}s^{n-1} + \dots c_0}{s^n + a_{n-1}s^{n-1} + \dots a_0}$$

Ex: $n=2$ max flat $v_o/v_i = \dfrac{1}{s^2+\sqrt{2}\,s+1}$ given

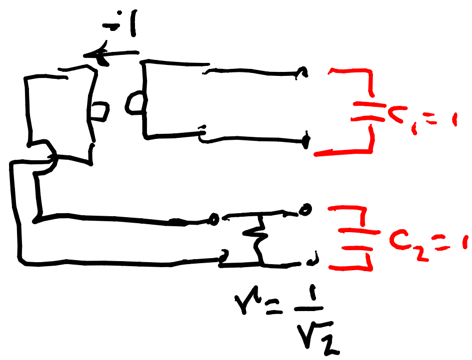
$$\begin{bmatrix}\dot x_1\\ \dot x_2\end{bmatrix} = \begin{bmatrix}0 & 1\\ -1 & -\sqrt{2}\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}0\\ 1\end{bmatrix}u \Rightarrow \begin{aligned}\dot x &= Ax + Bu\\ y &= Cx + Du + E\dot u\end{aligned}$$

$$y = \begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}$$

if $$-\begin{bmatrix}i_1\\ i_2\end{bmatrix} = \begin{bmatrix}C_1 & 0\\ 0 & C_2\end{bmatrix}\begin{bmatrix}\dot v_1\\ \dot v_2\end{bmatrix} = \begin{bmatrix}\dot x_1\\ \dot x_2\end{bmatrix} = \begin{bmatrix}0 & 1\\ -1 & -\sqrt{2}\end{bmatrix}\underset{\begin{bmatrix}v_1\\ v_2\end{bmatrix}}{\underset{\parallel}{\begin{bmatrix}x_1\\ x_2\end{bmatrix}}} + \begin{bmatrix}0\\ 1\end{bmatrix}\overset{v_i}{\overset{\parallel}{u}}$$



but need inputs & outputs; $Y_{2port} = \begin{bmatrix}0 & -1\\ 1 & \sqrt{2}\end{bmatrix}$



$$\underset{\text{assume a current}}{\begin{bmatrix}i_1\\ i_2\\ \underset{\parallel}{y}\end{bmatrix}} = \begin{bmatrix} & Y_{3\times3} & \end{bmatrix}\begin{bmatrix}v_1\\ v_2\\ u\end{bmatrix} = v_i$$

$$\begin{bmatrix}-\dot x\\ \hline y\end{bmatrix} = \left[\begin{array}{c|c}-A & -B\\ \hline C & D\end{array}\right]\begin{bmatrix}x\\ \hline \underset{v_i}{\underset{\parallel}{u}}\end{bmatrix} \Rightarrow Y_{3\times3} = \left[\begin{array}{cc|c}0 & -1 & 0\\ 1 & \sqrt{2} & 1\\ \hline 1 & 0 & 0\end{array}\right] = \text{constant, make with OTA, resistors}$$

$i_0 = v_0$