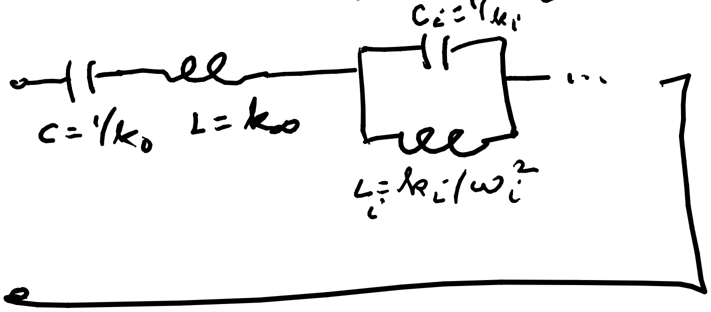
EE 610
10/15/13

Note Title 10/15/2013

RC $\leftrightarrow$ LC

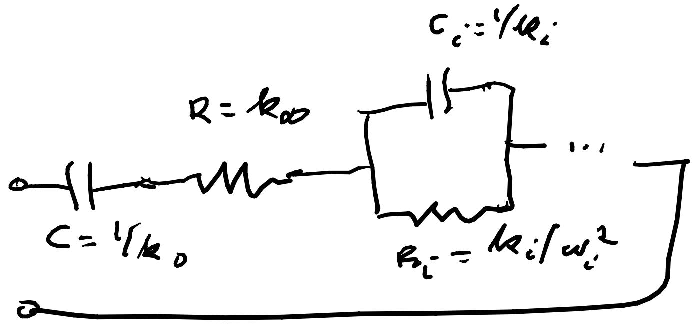
1st Foster

$$Z_{LC}(s) = \frac{k_0}{s} + k_\infty s + \sum_{i=1}^{m} \frac{k_i s}{s^2+\omega_i^2}$$



$$Y_i = \frac{1}{\frac{s}{k_i} + \frac{\omega_i^2}{k_i s}}$$

$\Rightarrow$ RC $\Longrightarrow$ Z_{RC}



$$Z_{RC}(s) = \frac{k_0}{s} + k_\infty + \sum_{i=1}^{m} \frac{1}{\frac{s}{k_i} + \frac{\omega_i^2}{k_i}} = \frac{k_0}{s} + k_\infty + \sum \frac{k_i}{s+\omega_i^2}$$

all poles are on $-\sigma$ axis, all residues positive
zeroes & poles alternate, possible pole @ $s=0$
none @ ∞

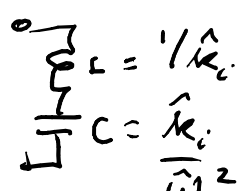
For $$Y_{LC} = \frac{\hat{k}_0}{s} + \hat{k}_\infty s + \sum_{i=1}^{\hat{m}} \frac{\hat{k}_i s}{s^2+\hat{\omega}_i^2}$$

$$Y_i = \frac{1}{\frac{s}{\hat{k}_i} + \frac{\hat{\omega}_i^2}{k_i s}}$$

for 2nd Foster

$$Y_{RC} = \hat{k}_0 + \hat{k}_\infty s + \sum_{i=1}^{\hat{m}} \frac{\hat{k}_i s}{s+\hat{\omega}_i^2}$$

$$Y_{i_{RC}} = \frac{1}{\frac{1}{\hat{k}_i} + \frac{\omega_i^2}{\hat{k}_i s}} \Leftarrow =$$



$$= \frac{\hat{k}_i s}{s+\hat{\omega}_i^2}$$

(is not a partial fraction expansion)

$$\frac{Y_{RC}}{s} = \frac{\hat{k}_0}{s} + \hat{k}_\infty + \sum_{i=1}^{\hat{m}} \frac{\hat{k}_i}{s+\hat{\omega}_i^2}\,;$$ this is a partial fraction expansion

∴ to synthesize from y_{RC} form $\frac{y_{RC}}{s}$ & do partial fraction expansion
Then multiply back by the s

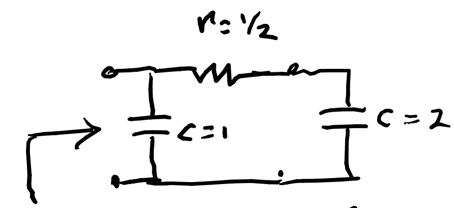
For 1st Cauer of RC remove poles of y_{RC} @ ∞ & constants at ∞ of Z_{RC}

Ex: $Z_{RC}(s) = \dfrac{(s+1)}{s(s+3)} = \dfrac{s+1}{s^2+3s} = \dfrac{1}{y_{RC}} = \cfrac{1}{\cfrac{s^2+3s}{s+1}}$

$$= \cfrac{1}{s + \cfrac{1}{\frac{1}{2} + \cfrac{1}{2s}}} \qquad \begin{array}{l} \leftarrow z \\ \leftarrow y \\ \leftarrow z \\ \leftarrow y \end{array}$$

$$\begin{array}{r l}
 & s \\
s+1 \,) & s^2+3s \\
 & s^2+s \\
 & \overline{2s} \,) \; s+1 \quad \tfrac{1}{2} \\
 & \qquad s \\
 & \qquad \overline{1} \,) \; 2s \quad 2s \\
 & \qquad\qquad 2s
\end{array}$$

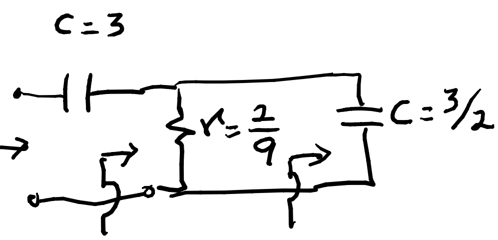
1st Cauer



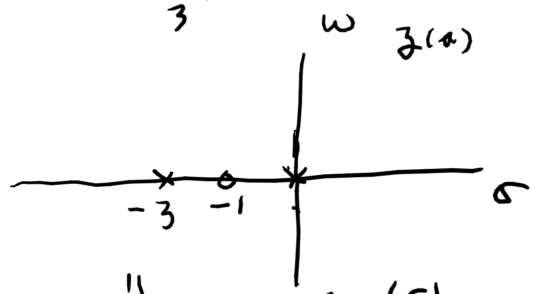
$Z = \dfrac{s+1}{s(s+3)}, \quad y = \dfrac{s^2+3s}{s+1}$

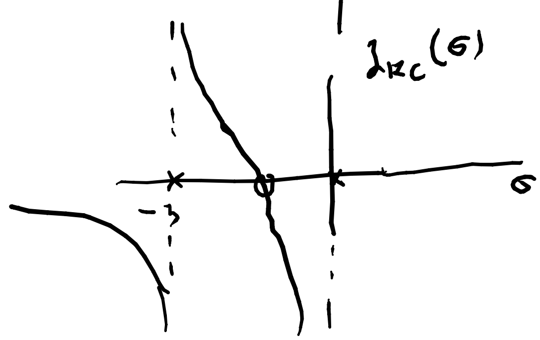
2nd Cauer

$$\begin{array}{r l l}
 & \frac{1}{3s} & \leftarrow z \\
3s+s^2 \,) & 1+s & \\
 & 1+s/3 & \\
 & \overline{\tfrac{2}{3}s} \,) \; 3s+s^2 \quad \tfrac{9}{2} & \leftarrow y \\
 & \quad 3s & \\
 & \quad \overline{s^2} \,) \; \tfrac{2}{3}s \quad \tfrac{2}{3s} & \leftarrow z \\
 & \qquad \tfrac{2}{3}s &
\end{array}$$



$Z = \dfrac{s+1}{s(s+3)}$



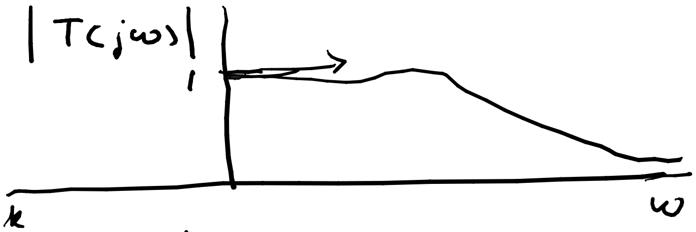


Maximally flat low pass $T(s) = \dfrac{A_0}{s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0}$
(at $\omega = 0$)

assume $a_i > 0$

normalize $a_0 = 1$ also $\Rightarrow$ $T(s) = \dfrac{1}{s^n + a_{n-1}s^{n-1} + \cdots a_1 s + 1}$

divide by A_0



$$\frac{d^k|T(j\omega)|}{d\omega^k} = 0 \quad , \text{ k as large as possible}$$

$$\frac{d\left|\frac{1}{T(j\omega)}\right|}{d\omega} = -\frac{1}{|T(j\omega)|^2}\cdot\frac{d|T(j\omega)|}{d\omega}$$

both 0 if $\frac{d|T(j\omega)|}{d\omega} = 0$
& if they are 0 then $\frac{d|T(j\omega)|}{d\omega} = 0$

also

$$\frac{d|T(j\omega)|^2}{d\omega} = 2|T(j\omega)|\cdot\frac{d|T(j\omega)|}{d\omega}$$

$$\frac{1}{T(j\omega)}\cdot\frac{1}{T^*(j\omega)} = \frac{1}{|T(j\omega)|^2} = \frac{1}{T(j\omega)\times T(-j\omega)} = \frac{1}{T(s)}\times\frac{1}{T(-s)}\Big|_{s=j\omega}$$

real coeff.

$$\frac{1}{T(s)}\times\frac{1}{T(-s)} = \left(s^n + a_{n-1}s^{n-1} + \cdots + 1\right)\left((-s)^n + a_{n-1}(-s)^{n-1} + \cdots + 1\right)$$

even in s, $s = j\omega \Rightarrow$ even in ω

$$\left|\frac{1}{T(j\omega)}\right|^2 = 1 + 0 + \cdots 0 + (j\omega)^n(-j\omega)^n = 1 + (+1)^n(-1)^n(j^2)^n\omega^{2n}$$

due to $\frac{d\,|1/T(j\omega)|}{d\omega}\Big|_{\omega=0} = 0$

$$= 1 + \omega^{2n}$$

$$= \frac{1}{T(s)}\cdot\frac{1}{T(-s)}\Big|_{s=j\omega}$$

$$\frac{1}{T(s)}\cdot\frac{1}{T(-s)} = 1 + \left(s/j\right)^{2n} = 1 + (-1)^n s^{2n} \qquad \text{set } \omega = s/j$$

to factor $(-1)^n s^{2n} + 1 = 0 \Rightarrow s^{2n} = (-1)^{n+1}$