EE610
10/10/13

Note Title 10/10/2013

PR lossless, $y(s) = -y(-s)$

Ex: $y(s) = \dfrac{(s^2+1)(s^2+9)}{s(s^2+4)} = \dfrac{s^4+10s^2+9}{s^3+4s}$

1st Foster = partial fraction expansion of $Z(s)$

2nd Foster = " " " of $y(s)$

1st Cauer = Continued fraction expansion about $s=\infty$

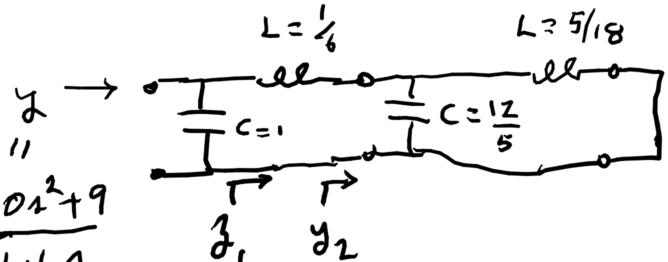
2nd Cauer = " " " about $s=0$

$$\begin{array}{r|l} s^3+4s & s^4+10s^2+9 \quad \rightarrow s \\ & s^4+4s^2 \\ \hline 6s^2+9 & s^3+4s \quad \rightarrow \tfrac{1}{6}s \\ & s^3+\tfrac{9}{6}s \\ \hline \tfrac{15}{6}s & 6s^2+9 \quad \rightarrow \tfrac{36}{15}s \\ & 6s^2 \\ \hline 9 & \tfrac{15}{6}s \quad \rightarrow \tfrac{15s}{6\times 9} \end{array}$$

$$y(s) = s + \underbrace{\frac{6s^2+9}{s^3+4s}}_{y_1}$$

$$= s + \cfrac{1}{\tfrac{1}{6}s + \underbrace{\cfrac{15s/6}{6s^2+9}}_{z_2}}$$

$$= s + \cfrac{1}{\tfrac{1}{6}s + \cfrac{1}{\tfrac{36}{15}s + \cfrac{1}{\tfrac{5}{18}s}}}$$



$y = \dfrac{s^4+10s^2+9}{s^3+4s}$

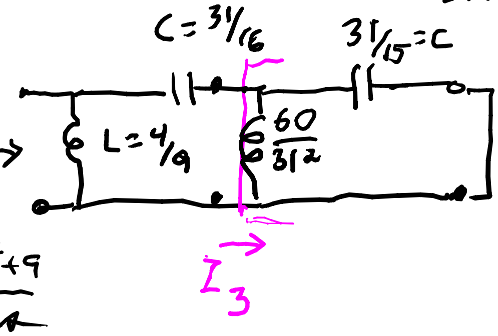
1st Cauer

For 2nd Cauer remove poles at $s=0$

$$\begin{array}{r|l} 4s+s^3 & 9+10s^2+s^4 \quad \rightarrow \tfrac{9}{4s} \\ & 9+\tfrac{9}{4}s^2 \\ \hline \tfrac{31}{4}s^2+s^4 & 4s+s^3 \quad \rightarrow \tfrac{16}{31s} \\ & 4s+\tfrac{16}{31}s^3 \\ \hline \tfrac{15}{31}s^3 & \tfrac{31}{4}s^2+s^4 \quad \rightarrow \tfrac{31^2}{60s} \end{array}$$

$$\frac{31}{4}s^2 \qquad \frac{15}{31s}$$
$$s^4 \,\Big)\, \frac{15}{31}s^3$$

$$y(s) = \overset{y_1}{\downarrow}\frac{9}{4s} + \cfrac{\overset{z_2}{\downarrow}\;1\;\overset{y_3}{\downarrow}\;\;\overset{z_4}{\downarrow}}{\frac{16}{31s} + \underbrace{\left[\cfrac{1}{\frac{31^2}{60s} + \cfrac{1}{\frac{15}{31s}}}\right]}_{= Z_3}}$$



$y(s)$ = $\dfrac{s^4+10s^2+9}{s^3+4s}$

2nd Cauer ($\Rightarrow$ high pass filters)

$$P(s) = s^5 + 3s^4 + 5s^3 + 2s^2 + 3s + 6$$

$$2\,\mathrm{Ev}\,P(s) = P(s) + P(-s) = 2\left[3s^4 + 2s^2 + 6\right]$$
$$2\,\mathrm{Od}\,P(s) = P(s) - P(-s) = 2\left[s^5 + 5s^3 + 3s\right]$$

if form $\dfrac{P(s)}{\mathrm{Ev}(s)} = 1 + \dfrac{\mathrm{Od}\,P(s)}{\mathrm{Ev}\,P(s)} = 1 + \dfrac{s(s^4+5s^2+3)}{3s^4+2s^2+6}$

then $P(s)$ is Hurwitz if $z(s) = \dfrac{\mathrm{Od}\,P}{\mathrm{Ev}\,P}$ is a reactance function

lossless PR

is $z(s) = \dfrac{s^5+5s^3+3s}{3s^4+2s^2+6}$ a reactance function

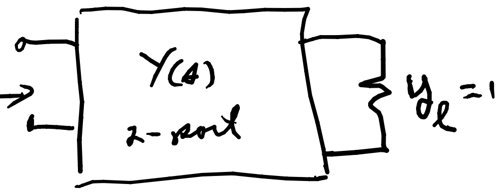
$$
\begin{array}{l}
3s^4+2s^2+6 \,\Big)\, s^5+5s^3+3s \quad \left(\tfrac{1}{3}s\right) \\
\qquad\qquad\quad s^5 + \tfrac{2}{3}s^3 + 2s \\
\qquad\qquad\quad \tfrac{13}{3}s^3 + s \,\Big)\, 3s^4+2s^2+6 \quad \left(\tfrac{9}{13}s\right) \\
\qquad\qquad\qquad\qquad\quad 3s^4 + \tfrac{9}{13}s^2 \\
\qquad\qquad\qquad\qquad\quad \tfrac{17}{13}s^2 + 6 \,\Big)\, \tfrac{13}{3}s^3 + s \quad \left(\tfrac{13^2}{3\times 17}s\right) \\
\qquad\qquad\qquad\qquad\qquad\qquad \tfrac{13}{3}s^3 + \tfrac{2\times 13^2}{17}s \\
\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \parallel
\end{array}
$$

remainder =

$1 - \frac{2 \times 13^2}{17} = \frac{17 - 26 \times 13}{17}$

< 0

shows that $P(s)$ is not Hurwitz $\Rightarrow$ a zero in $\sigma > 0$



$y(s) = \frac{y_{11} y_\ell + \Delta y}{y_\ell + y_{22}} = \frac{y_{11} + \Delta y}{1 + y_{22}}$

if 2-port is lossless $\Rightarrow$ y_{22} & y_{11} are reactance functions

here $1 + y_{22}$ should give a stable circuit $\Rightarrow$ $y_{22} = \frac{N}{D} = \frac{N + D}{D} \Rightarrow$

$N + D =$ Hurwitz