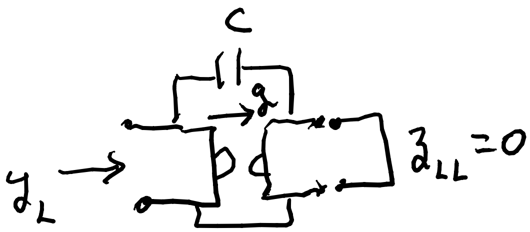
EE 610
10/03/13

Ex $y_L = C_L s$: $\frac{y_{LL}}{y_L(k)} = \frac{k\, y_L(k) - s\, y_L(s)}{k\, y_L(s) - s\, y_L(k)} = \frac{k C_L k - s^2 C_L}{k C_L s - s C_L k}$

$= \frac{C_L(k^2 - s^2)}{C_L(ks - ks)} \Rightarrow \frac{1}{y_{LL}} = 0 = z_{LL}$



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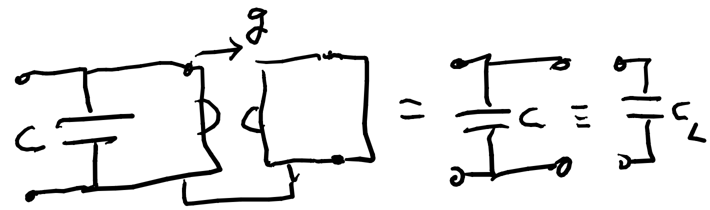
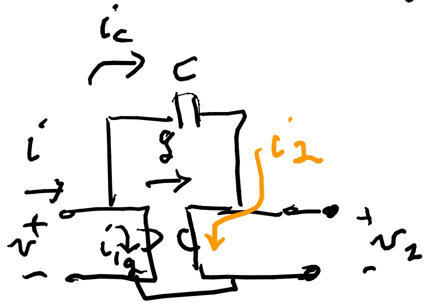


Ex 2: $y = \frac{1}{Ls}$

$\frac{y_L}{y(k)} = \frac{k\, y(k) - s\, y(s)}{k\, y(s) - s\, y(k)} = \frac{k\frac{1}{kL} - s\cdot\frac{1}{Ls}}{k\cdot\frac{1}{Ls} - s\cdot\frac{1}{Lk}} = \frac{\frac{1}{L}(1-1)}{\frac{1}{kLs}(k^2 - s^2)} = 0$



$i = i_1 + i_2$ of the gyrator

$= g v_2 - g v_1 = g(v_2 - v_1)$

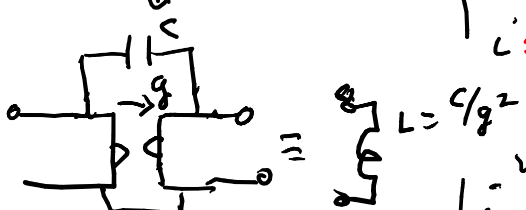
$i_c = Cs(v_1 - v_2) = i_2 \Rightarrow v_2 - v_1 = \frac{-i_2}{Cs}$

$i = -g\cdot\frac{i_2}{Cs}$ but $i_2 = -g v_1 = -g v$

$\Rightarrow i = -\frac{g(-gv)}{Cs} = \frac{g^2}{Cs}\cdot v$

$g v_1 = i$

$\Downarrow$



$\Downarrow$

$\boxed{i = +\frac{g^2}{Cs} v}$

Passive circuit

$P_{in}(t) = v^{*T}(t)\cdot i(t)$, Passive if $\underset{\text{energy in}}{E(t)} = \int_{-\infty}^{t} p(\tau)\,d\tau$, $-\infty < t$

≥ 0 (passive

$\Longleftrightarrow \int_{-\infty}^{\infty} p(\tau)\,d\tau \geq 0$

$$\int_{-\infty}^{\infty} v^{*T}(t)\, i(t)\, dt = \int_{-\infty}^{\infty} V^{*T}(j2\pi f) I(j2\pi f)\, df$$ *Parseval's theorem*

$$I(j\omega) = I(j2\pi f) = \int_{-\infty}^{\infty} i(t) e^{-j2\pi f t}\, dt = \text{Fourier transform}$$

$$\Rightarrow \int_{-\infty}^{\infty} V^{*T}(j\omega)\, Y(j\omega) V(j\omega)\, d\frac{\omega}{2\pi} \geq 0$$ for passivity

$$\Rightarrow V^{*T} Y(j\omega) V \Rightarrow \frac{V^{*T} Y(j\omega) V + V^T Y^T(j\omega) V^*}{2}$$

work with $\dfrac{V^{*T} Y(j\omega) V + V^{T*} Y^{T*}(j\omega) V}{2} = \mathrm{Re}\, V^{*T} Y(j\omega) V \geq 0$

(as $v(t) = v^*(t)$)

$$\Rightarrow \underbrace{\frac{V^{*T}\left(Y(j\omega) + Y^{T*}(j\omega)\right)V}{2}}_{} \geq 0$$

$$Y_{hermetian} = \frac{Y + Y^{T*}}{2} \Rightarrow Y_{hermetian}(j\omega) \geq 0$$

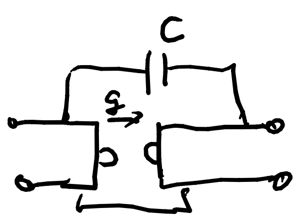
$\parallel$ $\mathrm{Re}\, y(j\omega)$ if Y is 1×1 matrix

Ex: $$Y(s) = \begin{bmatrix} cs & -cs+g \\ -cs-g & cs \end{bmatrix}$$

$$\frac{Y(j\omega) + Y^T(j\omega)}{2} = \frac{1}{2}\left\{ \begin{bmatrix} j\omega c & -j\omega c+g \\ -j\omega c-g & j\omega c \end{bmatrix} + \begin{bmatrix} -j\omega c & j\omega c-g \\ +j\omega c+g & -j\omega c \end{bmatrix} \right\}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \Rightarrow [V_1^* \; V_2^*] \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \geq 0$$

for all $\begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$

$\Rightarrow$ 

passive if g & $C \geq 0$ are real

Positive real needs $Y(s) + Y^{T*}(s) \geq 0$ in $\sigma > 0$

Ex: $y(s) = cs \Rightarrow c(\sigma + j\omega) + c(\sigma - j\omega) = 2c\sigma \geq 0$ in $\sigma > 0$

$\Rightarrow c \geq 0$ for PR

$V^{*T} Y(s) V$ has no poles in $\sigma > 0$ if rational, $s = \sigma + j\omega$

$e^{-(V^{*T} Y(s) V)}$ also is analytic in $\sigma > 0$

$$\left| e^{-(V^{*T} Y(s) V)} \right| = \left| e^{-\mathrm{Re}(V^{*T} Y(s) V)} \right| \underbrace{\left| e^{-j\,\mathrm{Im}(V^{*T} Y(s) V)} \right|}_{1} = e^{-\mathrm{Re}(V^{T*} Y(s) V)}$$

by maximum modulus theorem $\left| e^{-(V^{T*} Y(s) V)} \right|$ is maximum on $j\omega$ axis for the region $\sigma > 0$

$\Rightarrow \quad \mathrm{Re}(V^{T*} Y(s) V) \geq 0$ in $\sigma > 0$

$$\frac{1}{2}\left(V^{T*} Y(s) V + V^{T*} Y^{T*}(s) V\right) = \frac{1}{2} V^{T*}\left(Y(s) + Y^{T*}(s)\right) V$$

≥ 0 for all V's in $\sigma > 0$

key of the positive real condition

Ex above: gyrator c

$$\frac{Y(s) + Y^{T*}(s)}{2} = \frac{1}{2}\begin{bmatrix} cs + c^* s^* & -cs - c^* s^* + g - g \\ -cs - c^* s^* + g - g & cs + c^* s^* \end{bmatrix} = c\sigma \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

1st condition is $Y(\sigma)$ is real $\Rightarrow c$ real

? for PR is $c\sigma \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ positive semi definite (≥ 0)

(is only if $c \geq 0$)

$$c\sigma \begin{bmatrix} V_1^* & V_2^* \end{bmatrix} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} V_1 \\ 2 \end{bmatrix} = c\sigma \begin{bmatrix} V_1^* & V_2^* \end{bmatrix} \begin{bmatrix} V_1 - V_2 \\ V_2 - V_1 \end{bmatrix}$$

$$= \left[V_1^* V_1 - V_1^* V_2 + V_2^* V_2 - V_1 V_2^* \right] c\sigma$$

check for all V_1 & V_2 as complex numbers

true as all principal minors are ≥ 0