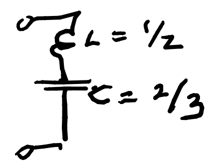
Note Title

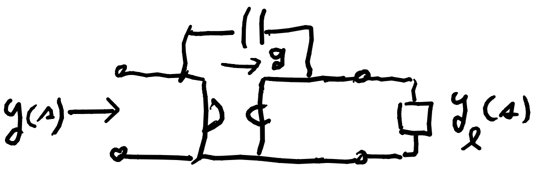
EE 610
10/01/13
even zero plot 10/03
9/27/2013

Synthesis: given $y(s)$, find a circuit

Ex: $y(s) = \dfrac{2s}{s^2+3}$; $z(s) = \dfrac{s^2+3}{2s} = \dfrac{s}{2} + \dfrac{3}{2s}$



look at an alternate C



$$y(s) = \frac{y_{11} \cdot y_\ell + \Delta y}{y_{22} + y_\ell}$$

solve for $y_\ell(s) \Rightarrow$ $y \cdot y_{22} + y \cdot y_\ell = y_{11} y_\ell + \Delta y$

$$(y - y_{11}) y_\ell = \Delta y - y_{22} \cdot y$$

$$y_\ell = \frac{\Delta y - y_{22} y}{y - y_{11}}$$

$$Y = \begin{bmatrix} sc & -sc+g \\ -sc-g & sc \end{bmatrix}$$; $\Delta y = (sc)(sc) - (-sc+g)(-sc-g)$
$= (sc)^2 - (sc^2 - g^2) = g^2$

$$y_\ell(s) = \frac{g^2 - sc \cdot y(s)}{y(s) - sc} = \frac{g^2\left(1 - \frac{sc}{g} \cdot \frac{y(s)}{g}\right)}{y(s) - sc}$$

$$\Rightarrow \frac{y_\ell(s)}{g} = \left(1 - \frac{sc}{g} \cdot \frac{y(s)}{g}\right) \Big/ \left(\frac{y(s)}{g} - \frac{sc}{g}\right)$$, fix s_0 = constant

$g = y(s_0)$; $s_0 = g/c$

$$\frac{y_\ell(s)}{y(s_0)} = \frac{\left(1 - \frac{s}{s_0} \cdot \frac{y(s)}{y(s_0)}\right)}{\frac{y(s)}{y(s_0)} - \frac{s}{s_0}}$$; look at $s = s_0$

$$\Rightarrow \frac{y_\ell(s_0)}{y(s_0)} = \frac{1-1}{1-1} \Rightarrow$$ a zero & a pole cancel

Look @ $s = s_1$

$$\frac{y_\ell(s_1)}{y(s_0)} = \left(1 - \frac{s_1}{s_0} \cdot \frac{y(s_1)}{y(s_0)}\right) \Big/ \left(\frac{y(s_1)}{y(s_0)} - \frac{s_1}{s_0}\right)$$

$$= \left(s_0\, y(s_0) - s_1\, y(s_1)\right) / \left(s_0\, y(s_1) - s_1\, y(s_0)\right)$$

desire $s_0\, y(s_0) = s_1\, y(s_1)$ & $s_0\, y(s_1) = s_1\, y(s_0)$

(for a 2nd cancellation)

$$\frac{y(s_0)}{y(s_1)} = \frac{y(s_1)}{y(s_0)} \Rightarrow y^2(s_0) = y^2(s_1)$$

$$\Rightarrow y(s_1) = \pm\, y(s_0)$$

$\therefore$ choose $y(s_1) = -y(s_0)$

$$s_0\, y(s_0) - s_1\, y(s_1) = s_0\, y(s_0) - s_1(-y(s_0))$$

$$= y(s_0)(s_0 + s_1)$$

desire then $s_1 = -s_0$

also $y(s_1) = -y(s_0) = -y(-s_1) \Rightarrow y(s_1) + y(-s_1) = 0$

given $y(s) \Rightarrow$ Even part is $y_{ev}(s) = \frac{1}{2}\left[y(s) + y(-s)\right]$

$\therefore$ if choose s_1 = zero of the even part of $y(s)$ then

$$(s - s_0)(s - s_1) = (s - s_0)(s + s_0) = s^2 - s_0^2$$

cancels. i.e. if s_0 is zero of the even part of $y(s)$ then in $y_\ell(s) = \dfrac{1 - \frac{s}{s_0}\frac{y(s)}{y(s_0)}}{\frac{y(s)}{y(s_0)} - \frac{s}{s_0}}$, $s^2 - s_0^2$ in numerator & denominator

so $y_\ell(s)$ has less degree by 1 of $y(s)$

$g = y(s_0)$, $C = g/s_0 = \dfrac{y(s_0)}{s_0}$ (really desire s_0 = real)

Ex: $y = \dfrac{2s}{s^2+3}$; $2\mathcal{E}\, y(s) = y(s) + y(-s) = \dfrac{2s}{s^2+3} + \dfrac{(-2s)}{(-s)^2+3} \equiv 0$

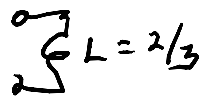
can choose any s_0, choose $s_0 = 1$, $s_1 = -s_0 = -1$

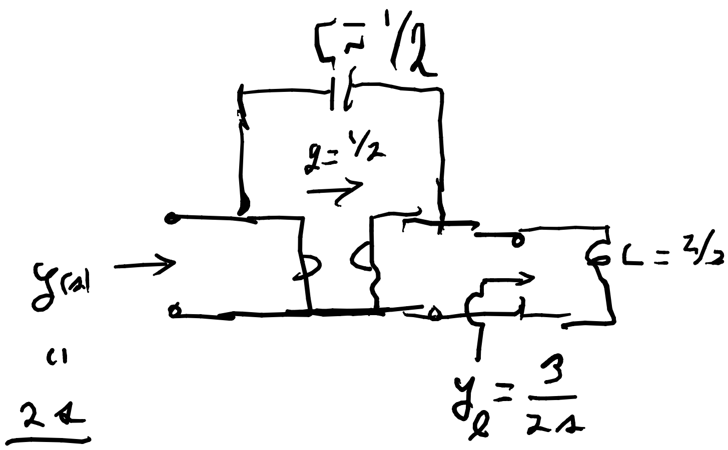
$$y(s_0) = g = \frac{2\times 1}{1^2+3} = 1/2, \quad C = g/s_0 = \frac{1/2}{1} = 1/2; \quad \frac{C}{g} = \frac{1/2}{1/2} = 1$$

$$\frac{y_\ell(s)}{y(s_0)} = 2y_\ell(s) = \frac{1 - \frac{Cs}{g}\cdot\frac{y(s)}{g}}{\frac{y(s)}{g} - \frac{Cs}{g}} = \frac{1 - s\cdot 2\left(\frac{2s}{s^2+3}\right)}{2\cdot\frac{2s}{s^2+3} - s}$$

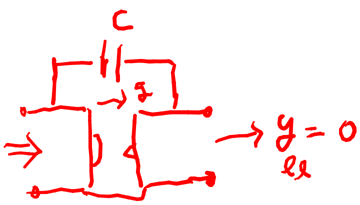
$$= \frac{(s^2+3)-4s^2}{4s-(s^3+3s)} = \frac{-3s^2+3}{-s^3+s} = \frac{-3(s^2-1)}{-s(s^2-1)} = \frac{3}{s}$$

$y_\ell = \frac{3}{2s} \Rightarrow$





can repeat on $y_\ell \Rightarrow$



used $\frac{y_\ell(s)}{y(s)} = \frac{k\,y(k) - s\,y(s)}{k\,y(s) - s\,y(k)}$ = Richard's function

takes PR functions into PR functions

PR = Positive-real & rational

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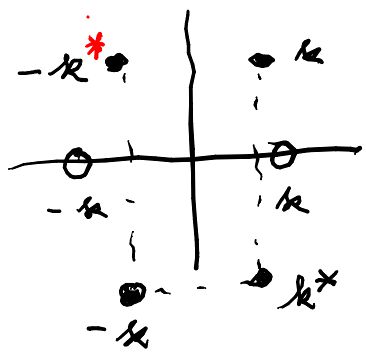
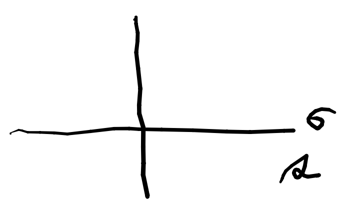
Eq. (8.6-1)



possible zeros of even

Positive Real, $f(s)$ or $F(s)$ as a matrix in $s = \sigma + j\omega$



1) Real for Real $s = \sigma > 0$ (real circuit)

2) analytic in Re $s = \sigma > 0$ (stable)

3) $F(s) + F^{T*}(s) \geq 0$ in $\sigma > 0$ (passive) (≥ 0 means positive semi-definite)

Ex: $Y(s) = \begin{bmatrix} cs & -cs+g \\ -cs-g & cs \end{bmatrix}$

1) $\Rightarrow$ c & g real

2) no finite pole $\Rightarrow$ analytic in $\sigma > 0$

3) form $Y(s) + Y^{T*}(s) = \begin{bmatrix} cs & -cs+g \\ -cs-g & cs \end{bmatrix} + \begin{bmatrix} cs^* & -g-cs^* \\ g-cs^* & +cs^* \end{bmatrix}$, $s = \sigma + j\omega$, $\sigma > 0$

$$= c\begin{bmatrix} s+s^* & -s-s^* \\ -s-s^* & s+s^* \end{bmatrix} = c(s+s^*)\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = 2\sigma c\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

if $c>0$ & in $\sigma>0$ need to check if $2\sigma c\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ is ≥ 0

here $\frac{\Delta_{11}}{2\sigma c} = 1 > 0$, $\frac{\Delta_{22}}{2\sigma c} = 1 > 0$, $\Delta = 0$ $\Leftrightarrow$ that $Y(s) + Y^{T*}(s) \geq 0$

$\therefore$ $Y(s)$ is a PR matrix for $c \geq 0$, g real