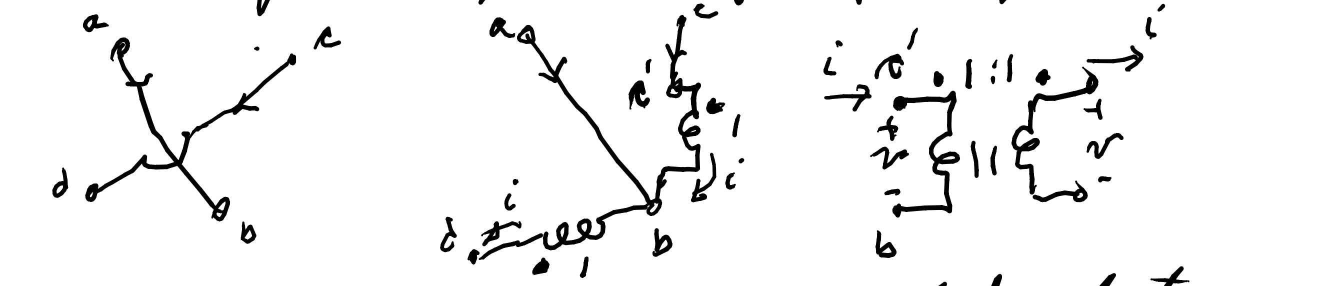
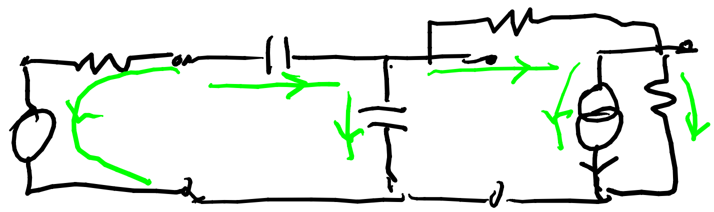
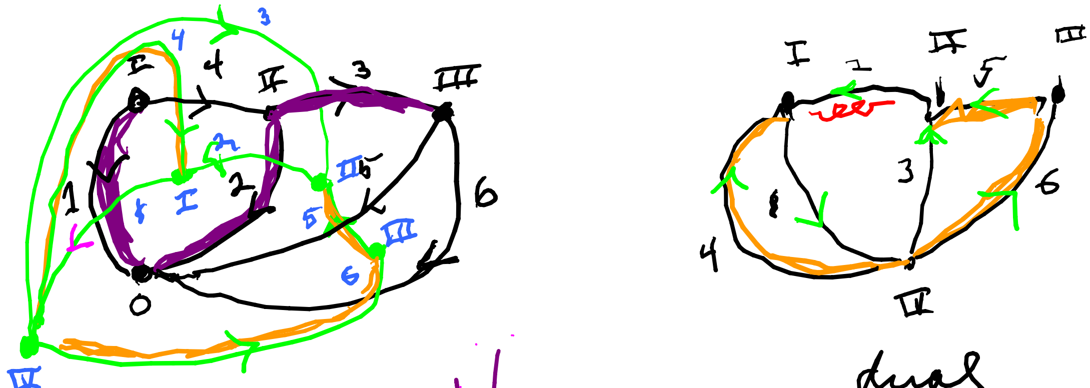
How to get a planar graph (for circuits)

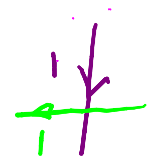


same i & v laws as before but no branches cross



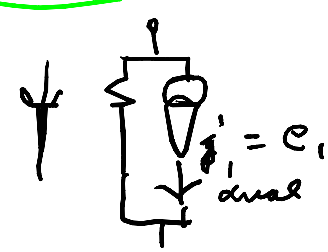


dual
(oriented by)



$$i = C\frac{dv}{dt} \overset{\text{dual}}{\Longrightarrow} v = C\frac{di}{dt} = L\frac{di}{dt}$$

$C = \text{const.}$



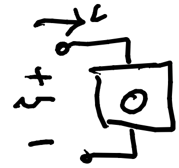
$G_{1_{\text{dual}}} = R_1$

if $C = C(t)$; $\quad i = \frac{dq}{dt} = \frac{d(Cv)}{dt} = C\frac{dv}{dt} + \frac{dC}{dt}\cdot v$

time varying g.

Norators & nullators:

Nullators:

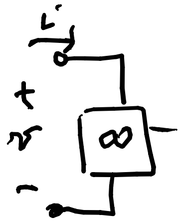


$i = 0$
$v = 0$

$Av = Bi$

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} v \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} i \end{bmatrix}$$

Norators:

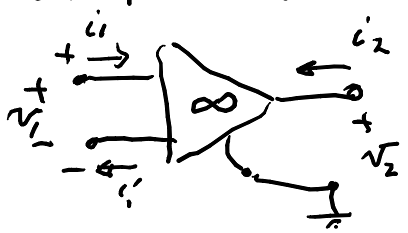


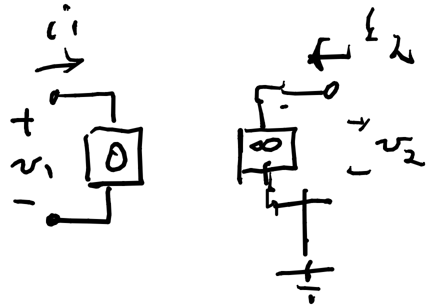
i = anything
v = anything independent of i

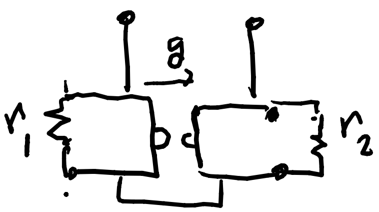
$Av = Bi$

$$\begin{bmatrix} 0 \end{bmatrix} v = \begin{bmatrix} 0 \end{bmatrix} i$$

ideal op-amp

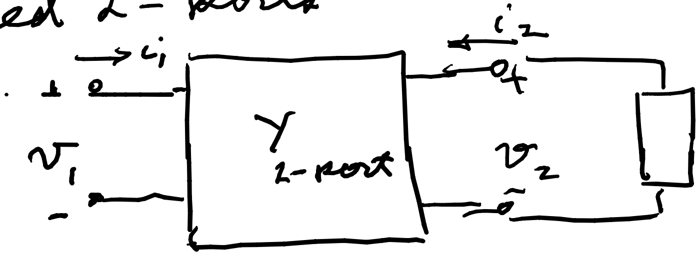






if $|r_1| = |r_2| = 1/g$
& one pos. & one neg
get a nullator or a norator

terminated 2-ports



$\Rightarrow i_2 = -y_L v_2$

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} i_1 \\ -y_L v_2 \end{bmatrix}$$

last row $y_{21} v_1 + (y_{22} + y_L) v_2 = 0 \Rightarrow v_2 = -(y_{22} + y_L)^{-1} y_{21} v_1$

1st row $\quad i_1 = y_{11} v_1 + y_{12}\left(-[y_{22} + y_L]^{-1} y_{21}\right) v_1$

$$i_1 = y_{in} v_1 \Rightarrow y_{in} = y_{11} - y_{12} [y_{22} + y_L]^{-1} y_{21}$$

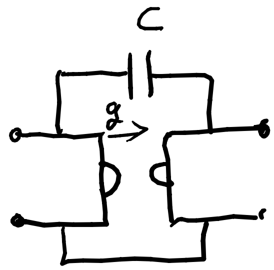
of interest is y_L in terms of y_{in}

$$-(y_{in} - y_{11}) = y_{12}[y_{22} + y_L]^{-1} y_{21} \quad \text{if } y_L \text{ is } 1\times 1$$

$$= (y_{22} + y_L)^{-1} y_{12} y_{21}$$

$$(y_{22} + y_L)^{-1} = \frac{y_{11} - y_{in}}{y_{12} y_{21}} \Rightarrow y_{22} + y_L = \frac{y_{12} y_{21}}{y_{11} - y_{in}}$$

$$\Rightarrow \quad y_L = -y_{22} + \frac{y_{12} y_{21}}{y_{11} - y_{in}} = \frac{y_{in} y_{22} - \Delta y}{y_{11} - y_{in}}, \quad \Delta y = y_{11} y_{22} - y_{21} y_{12}$$



$$Y = \begin{bmatrix} sC & -sC + g \\ -sC - g & sC \end{bmatrix}; \quad \Delta_y = (sC)^2 - (s^2C^2 - g^2) = g^2$$

$$y_L = \frac{sC\, y_{in} - g^2}{sC - y_{in}} =$$

use the Richards' function

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