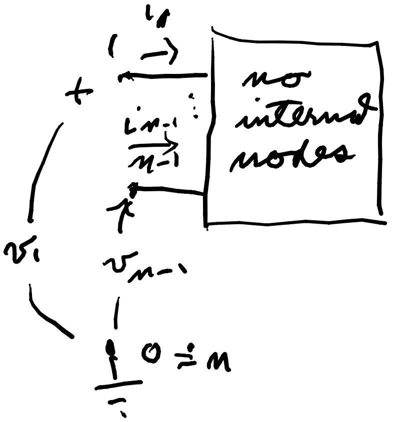
EE610
09/24/13



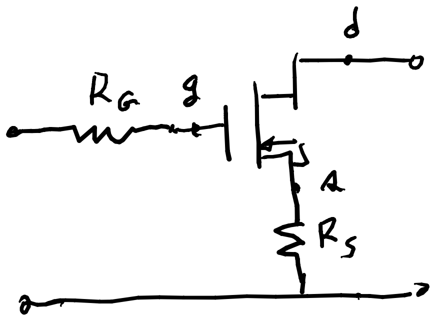
$$\underset{n-1}{i} = \underset{ind}{Y}\, \underset{1n-1}{v}$$

$$\sum_{j=1}^{n-1} i_j = 0 \Rightarrow \sum_i \text{ of } Y_{ij\,ind} = 0 \text{ for a fixed } j$$

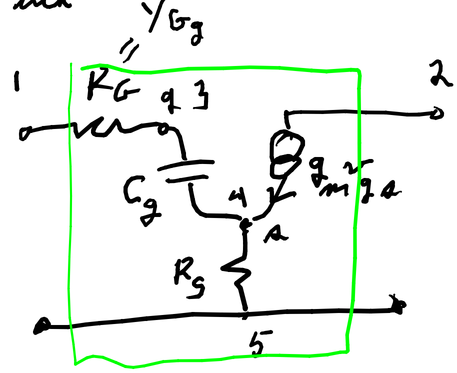
and also by adding E to each v_j,

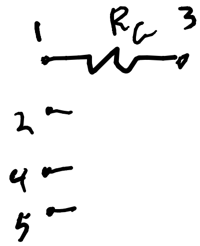
$$\sum_j Y_{ij\,ind} = 0 \text{ for a fixed } i$$

Ex: Y_{ind}



$\equiv$

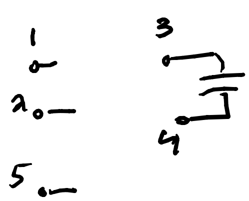




$\Rightarrow Y_{R_G} \Rightarrow \quad i_1 = G_g(v_1 - v_3) \quad i_3 = G_g(v_3 - v_1) \Rightarrow$

$$Y_{R_G} = \begin{bmatrix} G_g & 0 & -G_g & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ -G_g & 0 & G_g & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

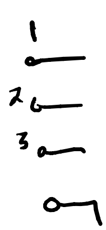
for C_g



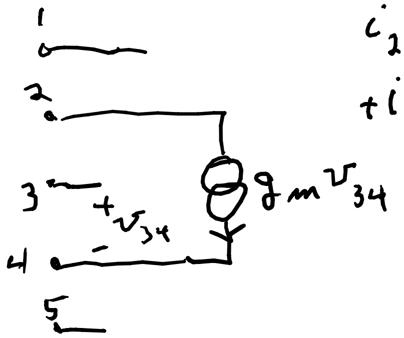
$$\Rightarrow Y_{C_g} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & sC_g & -sC_g & 0 \\ 0 & 0 & -sC_g & sC_g & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$ is indefinite

$$i_3 = sC_g(v_3 - v_4)$$
$$i_4 = sC_g(v_4 - v_3)$$

for $R_S = 1/G_S$



$$Y_{G_S} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & G_S & -G_S \\ 0 & 0 & 0 & -G_S & G_S \end{bmatrix}$$



$$i_2 = g_m(v_3 - v_4)$$
$$+i_4 = -g_m(v_3 - v_4) = g_m(-v_3 + v_4)$$

$$Y_{g_m} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & g_m & -g_m & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -g_m & g_m & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$Y_{ind} = Y_{G_g} + Y_{G_s} + Y_{C_g} + Y_{g_m} = \begin{bmatrix} G_g & 0 & -G_g & 0 & 0 \\ 0 & 0 & g_m & -g_m & 0 \\ -G_g & 0 & G_g + sC_g & -sC_g & 0 \\ 0 & 0 & -sC_g - g_m & G_s + sC_g + g_m & -G_s \\ 0 & 0 & 0 & -G_s & +G_s \end{bmatrix}$$

To get the 2-port Y
set $i_3 = 0$ & $i_4 = 0$,
1st move ground to node 5; sets $v_5 = 0$ ($\Rightarrow$ ignore column 5
& i_5 is = sum of other i's so
ignore row 5

$$\underset{(5=\text{gnd})}{Y_{def}} = \left[\begin{array}{cc|cc} G_g & 0 & -G_g & 0 \\ 0 & 0 & g_m & -g_m \\ \hline -G_g & 0 & G_g + sC_g & -sC_g \\ 0 & 0 & -sC_g - g_m & G_s + sC_g + g_m \end{array}\right] ; \quad \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} = \left[\begin{array}{c|c} Y_{11} & Y_{12} \\ \hline Y_{21} & Y_{22} \end{array}\right] \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix}$$

$0 = \{ i_3, i_4$

$$\underline{0}_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} i_3 \\ i_4 \end{bmatrix} = Y_{21} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} + Y_{22} \begin{bmatrix} v_3 \\ v_4 \end{bmatrix}$$

solve for $\begin{bmatrix} v_3 \\ v_4 \end{bmatrix} = Y_{22}^{-1}(-Y_{21}) \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \Rightarrow \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = (Y_{11} - Y_{12} Y_{22}^{-1} Y_{21}) \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$

$$= Y_{2\text{-port}} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$$Y_{2\text{-port}} = Y_{11} - Y_{12} Y_{22}^{-1} Y_{21}$$

$$= \begin{bmatrix} G_g & 0 \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} -G_g & 0 \\ g_m & -g_m \end{bmatrix} \begin{bmatrix} G_g + sC_g & -sC_g \\ -sC_g - g_m & G_s + sC_g + g_m \end{bmatrix}^{-1} \begin{bmatrix} -G_g & 0 \\ 0 & 0 \end{bmatrix}$$

Duals: $v \to i_d$ = dual $\qquad i = C\,dv/dt$

$i \to v_d \qquad v_d = C\,di_d/dt$

$$Y = \begin{bmatrix} 0 & 0 \\ g_m & 0 \end{bmatrix} \Rightarrow Z_d = \begin{bmatrix} 0 & 0 \\ g_m & 0 \end{bmatrix} \Rightarrow$$ voltage controlled by a current

dual of a graph $\Rightarrow$ problem only planar graphs have dual graph

— dual branches

— dual tree