EE610
09/17/13

Note Title 9/17/2013

$$E\dot{x} = Ax + Bu$$
$$y = Cx$$

$\Rightarrow$ transform to

$$\begin{bmatrix} 1_k & 0 \\ 0 & 0_{n-k} \end{bmatrix} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} \hat{A}_{11} & \hat{A}_{12} \\ \hat{A}_{21} & \hat{A}_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} \hat{B}_1 \\ \hat{B}_2 \end{bmatrix} u$$

"$\hat{E}$"

$$y = [\hat{C}_1 \ \hat{C}_2] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

if $\hat{A}_{22}$ is nonsingular
can eliminate nondynamics (last n-k rows of $\hat{E}$)

$$\dot{x}_1 = \hat{\hat{A}} x_1 + \hat{\hat{B}}_1 u$$
$$y = \hat{\hat{C}} x_1 + \hat{\hat{D}} u$$

If desire output vs input transfer function

$$E s X(s) = A X(s) + B U(s)$$
$$Y(s) = C X(s)$$

solve for $X(s)$: $(Es - A) X(s) = B U(s)$

assume $Es - A$ is nonsingular

$$X(s) = (Es - A)^{-1} B U(s)$$

$$Y(s) = C X(s) = C (Es - A)^{-1} B U(s)$$

$T(s) = C(Es - A)^{-1} B$ is the transfer function matrix

if u is an n-vector & y is an m vector this is a $m \times n$ matrix; in state variable case $n \to k$

$$T(s) = C(s1_k - A)^{-1} B + D$$

Ex: $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \dot{x} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$, $y = [c_1 \ c_2] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

$$Es-A = \begin{bmatrix} -1 & s \\ 0 & -1 \end{bmatrix}; \quad \det(Es-A) = 1$$

$$(Es-A)^{-1} = \frac{1}{1}\begin{bmatrix} -1 & -s \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} -1 & -s \\ 0 & -1 \end{bmatrix}$$

$$T(s) = [c_1 \;\; c_2]\begin{bmatrix} -1 & -s \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = [c_1 \;\; c_2]\begin{bmatrix} -s \\ -1 \end{bmatrix} = -(c_1 s + c_2)$$

Let $c_1 = -1$, $c_2 = 0 \Rightarrow T(s) = s$ (a differentiator)

one problem: given $T(s)$ find a circuit (synthesis)

In theory so far have $Av = Bi$

if B^{-1} is nonsingular then $i = B^{-1}Av$ is a good description

$B^{-1}A = Y_{b\times b}$ an admittance, branch by branch

assume $Y_{b\times b}$ exists $\Rightarrow$ $i = Y_{b\times b}v$

$$\begin{aligned} i_b &= i + j \\ v_b &= v + e \end{aligned} \Rightarrow i_b - j = Y_{b\times b}(v_b - e) \Rightarrow i_b = Y_{b\times b}v_b + (\underbrace{j - Y_{b\times b}e}_{\text{Norton (branch equivalents)}})$$

$\underline{0}_t = \mathcal{C} i_b$, $v_b = \mathcal{C}^T v_t$

$\underline{0}_\ell = \mathcal{J} v_b$ $\quad i_b = \mathcal{J}^T i_\ell$ $\quad \Rightarrow \mathcal{C} i_b = \underline{0}_t$

$$= \mathcal{C} Y_{b\times b}\mathcal{C}^T v_t + (\underbrace{\mathcal{C}j - \mathcal{C}Y_{b\times b}e}_{\text{Norton's equiv. in tree}})$$

$P_{in} = 0 = v_b^T i_b = v_t^T \mathcal{C}\mathcal{J}^T i_\ell$

$\Rightarrow \mathcal{C}\mathcal{J}^T = 0_{t\times \ell}$

here have t eqs in unknown v_t

$$(\mathcal{C}Y_{b\times b}e - \mathcal{C}j) = \mathcal{C}Y_{b\times b}\mathcal{C}^T v_t$$

if $(\mathcal{C}Y_{b\times b}\mathcal{C}^T)$ is nonsingular then a solution is

$$v_t = [\mathcal{C}Y_{b\times b}\mathcal{C}^T]^{-1}(\mathcal{C}Y_{b\times b}e - \mathcal{C}j)$$