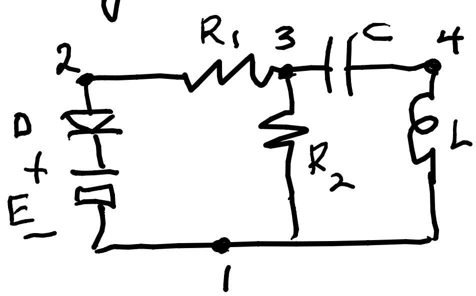
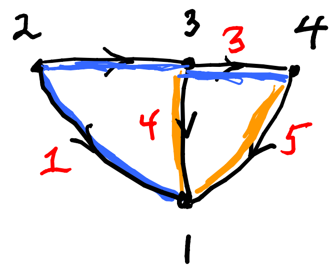
Note Title 9/5/2013

09/05/13
EE610

For next week see if you can get PSpice especially look @ Gvalue = VCCS (gain = any function)
& PARAM

Graphs of a network





• = node, # = n = 4

—— = branches, # b = 5

oriented branch →

$\overrightarrow{i}$
$+ \; v \; -$

$v \times i$ = power into branch component

—— = tree branches

= t = 3 = n-1

—— = co-tree = link

= l

$\Rightarrow b - l = t = n - 1$

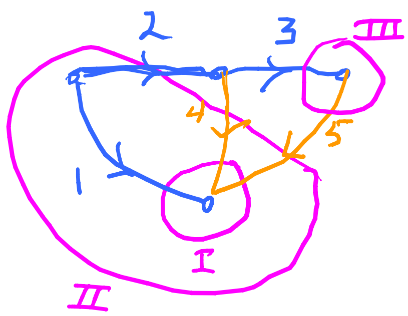
$t = \#$ of KCL $= n-1$

$l = \#$ of KVL $= b - t = b - (n-1)$

KCL = Σ currents into a sphere $\Rightarrow$ circle in projection onto the plane
= 0

do for each tree branch

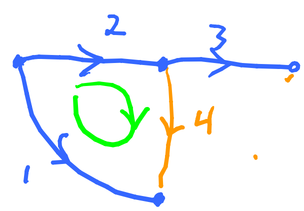
cut sets
$\mathcal{C}$



$$\begin{array}{c} \mathrm{I} \\ \mathrm{II} \\ \mathrm{III} \end{array}\begin{bmatrix}0\\0\\0\end{bmatrix} = \left[\begin{array}{ccc|cc} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & -1 & -1 \\ 0 & 0 & 1 & 0 & -1 \end{array}\right]\begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \end{bmatrix} \quad \} \, i_t$$

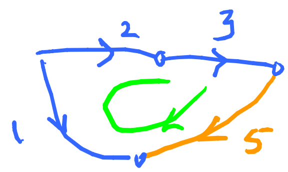
$\underline{0}_3 = \underline{0}_t$ $\quad$ $\mathbb{1}_3 = \mathbb{1}_t$ $\quad$ K

KVL for link 1



Tie sets
$\mathcal{T}$

KVL link 2



$$\begin{array}{c} \text{link } 1 \\ \text{link } 2 \end{array}\begin{bmatrix}0\\0\end{bmatrix} = \left[\begin{array}{ccc|cc} -1 & 1 & 0 & 1 & 0 \\ -1 & 1 & 1 & 0 & 1 \end{array}\right]\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{bmatrix} \quad \} \, i_\ell$$

$\underline{0}_2 = \underline{0}_\ell$ $\quad$ $-K^T$ $\quad$ $\mathbb{1}_2 = \mathbb{1}_\ell$

V_T = transpose

KCL $\quad \underline{0}_t = \mathcal{C} \cdot i_b \qquad \mathcal{C}$ is $t \times b$, cut set matrix, $\{0, 1, -1\}$ entries

KVL $\quad \underline{0}_\ell = \mathcal{T} \cdot v_b \qquad \mathcal{T}$ is $\ell \times b$, tie set matrix, $\{0, 1, -1\}$ entries

$$P_{in}(t) = v_b^T \cdot i_b = 0$$

To see: given v_t we know all voltages, so assume

$$v_b = M_v \cdot v_t, \quad M \text{ is } b \times t; \quad i_b = M_i \cdot i_\ell$$

$$v_b = \begin{bmatrix} v_t \\ v_\ell \end{bmatrix} = \begin{bmatrix} \mathbb{1}_t \\ \hline M_{21} \end{bmatrix} v_t \qquad i_b = \begin{bmatrix} i_t \\ i_\ell \end{bmatrix} = \begin{bmatrix} M_{11} \\ \hline \mathbb{1}_\ell \end{bmatrix} i_\ell$$

$$P_{in} = v_b^T \cdot i_b = v_t^T \left[\mathbb{1}_t \mid M_{21}^T \right] \cdot \begin{bmatrix} M_{11} \\ \mathbb{1}_\ell \end{bmatrix} i_\ell = v_t^T \underbrace{\left[M_{11} + M_{21}^T \right]}_{= 0_{t \times \ell}} i_\ell$$

$= 0$
(if finite circuit)

$$\Longrightarrow M_{11} = -M_{21}^T$$

as v_t can be arbitrarily chosen as can be i_ℓ

$$\Rightarrow \text{gives } v_b = \mathcal{C}^T \cdot v_t, \quad i_b = \mathcal{T}^T i_\ell$$

(i.e. this is for a generic graph)

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ \hline 1 & -1 & 0 \\ 1 & -1 & -1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

if $$\mathcal{C} = \left[\begin{array}{ccc|cc} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & -1 & -1 \\ 0 & 0 & 1 & 0 & -1 \end{array}\right]$$

$$\underbrace{}_{1_3} \qquad K$$

$$\mathcal{C}^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -1 & 0 \\ 1 & -1 & -1 \end{bmatrix}$$

Now know get t independent KCL eqs & l independent KVL eqs as rank $\mathcal{C} = t$, rank $\mathcal{T} = l$