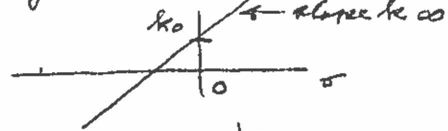
EE 610 F2013 final solutions

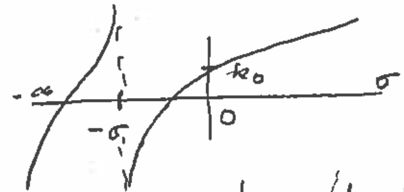
1\. a) $y(s) = k_\infty s + k_0 + \underbrace{\sum_{i=1}^{n-1} \frac{k_i s}{s+\sigma_i}}_{\text{absent if } n=1}$; $k_i, \sigma_i > 0$ for $i = 1, \dots, n-1, \infty$; $k_0 \geq 0$

$\sigma_i < \sigma_{i+1}$ $\qquad \frac{dy(\sigma)}{d\sigma} > 0$

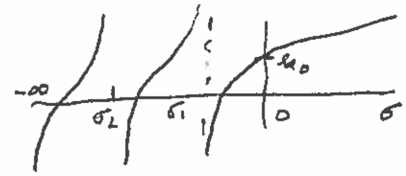
b) $n=1$; $y(\sigma) = k_\infty \sigma + k_0$



$n=2$: $y(\sigma) = k_\infty \sigma + k_0 + \frac{k_1 \sigma}{\sigma + \sigma_1}$



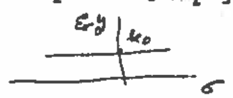
$n=3$: $y(\sigma) = k_\infty \sigma + k_0 + \frac{k_1 \sigma}{\sigma+\sigma_1} + \frac{k_2 \sigma}{\sigma + \sigma_2}$



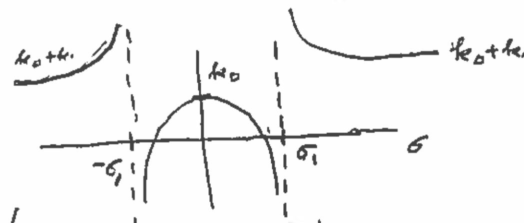
c) $Ev[y(\sigma)] = \frac{1}{2}(y(\sigma) + y(-\sigma))$; k_∞ cancels

$$= k_0 + \frac{1}{2}\sum_{i=1}^{n-1}\left[\frac{k_i\sigma}{\sigma+\sigma_i} + \frac{-k_i\sigma}{-\sigma+\sigma_i}\right] = k_0 + \sum_{i=1}^{n-1} \frac{-k_i \sigma^2}{(-\sigma^2+\sigma_i^2)}$$

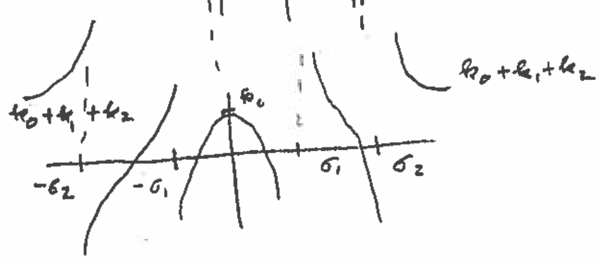
$n=1$ $\quad Ev\, y(\sigma) = k_0 \geq 0$



$n=2$ $\quad Ev\, y(\sigma) = k_0 - \frac{k_1\sigma^2}{(-\sigma+\sigma_1)(\sigma+\sigma_1)}$

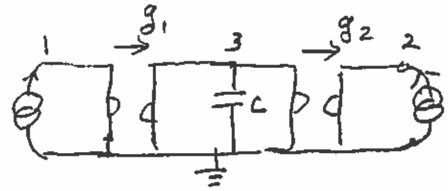
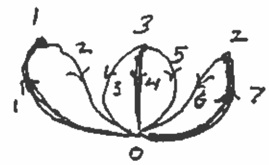


$n=3$ $\quad Ev\, y(\sigma) = k_0 - \frac{k_1\sigma^2}{(-\sigma+\sigma_1)(\sigma+\sigma_1)}$



If $k_0 = 0$ there is no real zero between $\sigma = 0$ & $\sigma = \sigma_1$ (In the case of $n=1$, $Ev\, y(\sigma) \equiv 0$)

d) If $k_0 > 0$ there is a finite real zero for all $n > 1$ but if $k_0 = 0$ not for $n=1$. ∴ Richards sections can proceed with real c-gyrator sections for all $n \geq 2$ and if $k_0 > 0$ also for $n=1$

2 a)  $\Rightarrow$ 

b) Cut set: $0_t = \mathcal{C} i_b \Rightarrow \mathcal{C} = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 1 \end{bmatrix}$

Tie set: $0_\ell = \mathcal{T} v_b \Rightarrow$

$$\mathcal{T} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

c) $Y_{ind}(s) = \left[\begin{array}{ccc|c} 0 & 0 & g_1 & -g_1 \\ 0 & 0 & -g_2 & g_2 \\ -g_1 & g_2 & sC & g_1 - g_2 - sC \\ \hline g_1 & -g_2 & -g_1 + g_2 - sC & sC \end{array}\right]$ (last row & column by sum of others)

d) grounding node 4 $\Rightarrow Y_{nodal}(s) = \begin{bmatrix} 0 & 0 & g_1 \\ 0 & 0 & -g_2 \\ -g_1 & g_2 & sC \end{bmatrix}$; to eliminate node 3 set $i_3 = 0$

$$i_3 = 0 \Rightarrow \begin{bmatrix} -g_1 & g_2 & sC \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = 0 \Rightarrow v_3 = -\frac{1}{sC} \begin{bmatrix} -g_1 & g_2 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} - \begin{bmatrix} g_1 \\ -g_2 \end{bmatrix} \frac{1}{sC} \begin{bmatrix} -g_1 & g_2 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$$\Rightarrow Y_{2port}(s) = \frac{1}{sC} \begin{bmatrix} g_1^2 & -g_1 g_2 \\ -g_2 g_1 & g_2^2 \end{bmatrix}$$ (note this is of rank 1 = degree 1 $\Rightarrow$ one L & a transformer)

e) $$y_{in} = y_{11} - y_{12} \frac{1}{y_{22} + y_L} \cdot y_{21} = \frac{\Delta y + y_{11} y_L}{y_{22} + y_L} = \frac{0 + \frac{g_1^2}{sC} \cdot sC_2}{\frac{g_2^2}{sC} + sC_2} = \frac{g_1^2}{sC_2 + g_2^2/sC}$$

$$= \frac{g_1^2 sC}{g_2^2 + (s^2 C C_2)}$$

f) This is RC only when either $C_2 = 0$ (or trivially if $C = 0$)

#3. a) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{x}_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -3 & 0 \\ 0 & -6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 3u_1 = 3u \\ 2u_2 = 2y_1 = 2x_1 \end{bmatrix}$; $y = y_2 = \begin{bmatrix} 0 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

which is

$$\begin{bmatrix} \dot{x}_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -3 & 0 \\ 2 & -6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 3 \\ 0 \end{bmatrix} u, \quad y = \begin{bmatrix} 0 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} ; \quad x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

b) $T(s) = \begin{bmatrix} 0 & 5 \end{bmatrix} \begin{bmatrix} s+3 & 0 \\ -2 & s+6 \end{bmatrix}^{-1} \begin{bmatrix} 3 \\ 0 \end{bmatrix}$; $(s1_2 - A)^{-1} = \frac{1}{(s+3)(s+6)} \begin{bmatrix} s+6 & 0 \\ 2 & s+3 \end{bmatrix}$

$$= 5 \cdot \frac{2}{(s+3)(s+6)} \cdot 3 = \frac{30}{(s+3)(s+6)}$$

c) $\begin{bmatrix} i = y \\ \left.\begin{matrix} -i_{c_1} \\ -i_{c_2} \end{matrix}\right\} = -\dot{x} \end{bmatrix} = \begin{bmatrix} 0 & C \\ -B & -A \end{bmatrix} \begin{bmatrix} u = v \\ \left.\begin{matrix} v_{c_1} \\ v_{c_2} \end{matrix}\right\} = v \end{bmatrix} = Y_c \begin{bmatrix} v \\ x \end{bmatrix}$

$\Rightarrow Y_c = \begin{bmatrix} 0 & 0 & 5 \\ -3 & 3 & 0 \\ 0 & -2 & 6 \end{bmatrix}$ $\Rightarrow$