Homework # 7 ENZE 610

Problem #1

- For odd n, roots will be as follows:

$$S_K = e^{j\frac{2k\pi}{2n}}$$

$$= e^{j\frac{k\pi}{n}}$$

$$S_K = e^{j\frac{k\pi}{3}} \qquad k = 0, \pm 1, \pm 2, \ldots \pm(2n-1)$$

$\therefore$

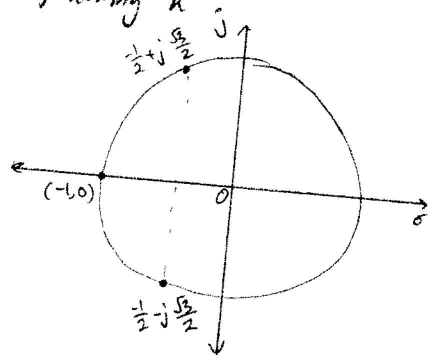
k	$\frac{k\pi}{3}$	
0	0	×
+ 1	$\pi/3$	×
− 1	$-\pi/3$	×
+ 2	$2\pi/3$	o
− 2	$-2\pi/3$	×
+ 3	π	o
− 3	$-\pi$	×
+ 4	$4\pi/3$	o
− 4	$-4\pi/3$	×
+ 5	$5\pi/3$	×
− 5	$-5\pi/3$	×

- We want roots in left half of complex plane for stability and for certain k, we obtain mirrored points
- Therefore we remain with the following k

$$k=2 \longrightarrow e^{j\frac{2\pi}{3}} = \frac{-1}{2} + j\frac{\sqrt{3}}{2}$$

$$k=3 \longrightarrow e^{j\pi} = -1$$

$$k=4 \longrightarrow e^{j\frac{4\pi}{3}} = \frac{-1}{2} - j\frac{\sqrt{3}}{2}$$



$$\therefore \frac{1}{T(s)} = \left(s - \left[\frac{-1}{2} + j\frac{\sqrt{3}}{2}\right]\right)\left(s - \left[\frac{-1}{2} - j\frac{\sqrt{3}}{2}\right]\right)\left(s - (-1)\right)$$

$$= s^3 + 2s^2 + 2s + 1$$

$$T(s) = \frac{1}{s^3 + 2s^2 + 2s + 1} \quad \text{(low pass)}$$

- For high pass, replace s with $\frac{1}{s}$:

$$T(s) = \frac{1}{(\frac{1}{s})^3 + \frac{2}{s^2} + \frac{2}{s} + 1}$$

$$T(s) = \frac{s^3}{s^3 + 2s^2 + 2s + 1} \quad \text{(high pass)}$$

Problem # 2

Low Pass:

$$T(s) = \frac{1}{s^3+2s^2+2s+1} = \frac{\frac{1}{(s^3+2s)}}{1+\frac{2s^2+1}{s^3+2s}} = \frac{-y_{21}}{1+\frac{y_{22}}{G_L}}$$

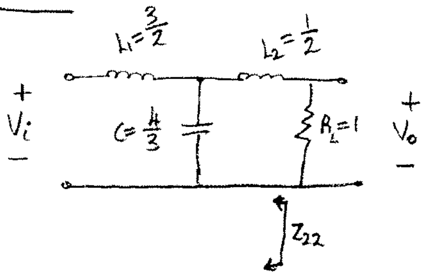
$$\frac{y_{22}}{G_L} = y_{22} = \frac{2s^2+1}{s^3+2s} \longrightarrow Z_{22} = \frac{s^3+2s}{2s^2+1}$$

1st Cauer:

$$\begin{array}{l} 2s^2+1 \,\big)\, s^3+2s \quad \left(\frac{s}{2}\right. \\ \quad s^3+\frac{s}{2} \\ \hline \quad \frac{3s}{2} \,\big)\, 2s^2+1 \quad \left(\frac{4s}{3}\right. \\ \quad 2s^2 \\ \hline \quad 1 \,\big)\, \frac{3s}{2} \quad \left(\frac{3s}{2}\right. \\ \quad \frac{3s}{2} \\ \hline \quad 0 \end{array}$$

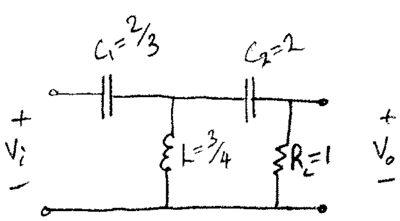
$$\Longrightarrow Z_{22} = \frac{s}{2} + \frac{1}{\frac{4s}{3}+\frac{1}{\frac{3s}{2}}}$$

So we obtain:



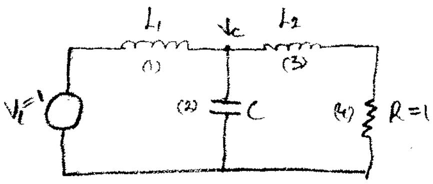
(low pass)

To get a high pass circuit we replace capacitors with inductors and vice versa:

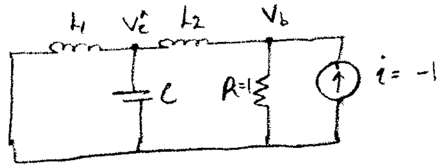


Problem #3

Original Circuit.



Adjoint Circuit.



$$\frac{dT(s)}{dx} = V_x^{aT}\,\frac{\Delta Y^T}{\Delta x}\cdot V_x \qquad \left[\text{change in } T(s) \text{ with respect to changes in component } x\right]$$

$$S = \frac{\frac{dT(s)}{dx}}{\frac{T(s)}{x}} = \frac{x}{T(s)}\cdot\frac{dT(s)}{dx}$$

$$i_b = Y\,V_b$$

$$i_b = \begin{bmatrix} \frac{1}{sL_1} & 0 & 0 & 0 \\ 0 & sC & 0 & 0 \\ 0 & 0 & \frac{1}{sL_2} & 0 \\ 0 & 0 & 0 & \frac{1}{R} \end{bmatrix} V_b$$

$\therefore$ if we look at C:

$$\frac{\Delta Y}{\Delta C} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & s & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

- The only non-zero element is at (2,2), so to compute $V_x^{aT}\frac{\Delta Y^T}{\Delta x}V_x$, we need only look at element (2) of V_x^T and V_x^a $\longrightarrow$ i.e. the voltage across component x

$\therefore$ from original circuit:

$$\frac{V_i - V_c}{sL_1} = \frac{V_c}{(sL_2+R)/\!/\frac{1}{sC}} \longrightarrow V_c = V_x^T = \frac{1}{s^2CL_1 + \frac{sL_1}{sL_2+1} + 1}$$

from adjoint circuit:

$$\left.\begin{aligned} i &= \frac{V_b}{R} + \frac{V_b}{sL_2 + sL_1/\!/\frac{1}{sC}} \\ \frac{V_b - V_c^a}{sL_2} &= \frac{V_c^a}{sL_1/\!/\frac{1}{sC}} \end{aligned}\right\} V_c^a = V_x^a = \frac{-sL_1}{s^3L_1L_2C + s^2L_1C + s(L_1+L_2) + 1}$$

$$= \frac{-3s}{2(s^3 + 2s^2 + 2s + 1)}$$

$$S = \frac{x}{T(s)}\cdot V_c\,\frac{\Delta Y}{\Delta x}\,V_c^a$$

$$S = \frac{-(s^3L_1L_2C + s^2L_1C)}{s^3L_1L_2C + s^2L_1C + s(L_1+L_2) + 1} = \frac{-(3s^3C + 6s^2C)}{3s^3C + 6s^2C + 8s + 4} = -\frac{s^3 + 2s^2}{s^3 + 2s^2 + 2s + 1}$$

Similarly for inductor L_1:

$$\left.\begin{aligned} \frac{\Delta Y}{\Delta L_1} &= \frac{-1}{sL_1^2} \\ V_L &= V_c - V_i = V_c - 1 \\ V_L^a &= V_c^a \end{aligned}\right\} S = \frac{s^3L_1L_2C + s^2L_1C + sL_1}{s^3L_1L_2C + s^2L_1C + s(L_1+L_2) + 1} = \frac{4s^3L_1 + 8s^2L_1 + 6sL_1}{4s^3L_1 + 8s^2L_1 + s(6L_1+3) + 6} = -\frac{2s^3 + 4s^2 + 3s}{2s^3 + 4s^2 + 4s + 2}$$