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610 Fall 20123– Homework 6 Due Th 10/24/13

1. (20 points) (Lossless Synthesis) Synthesize by the two Cauer and the two Foster forms the lossless admittances and compare the results.

$$y_1(s) = \frac{s(s^2+4)}{(s^2+1)(s^2+5)}$$

$$y_2(s) = \frac{(s^2+1)(s^2+5)}{s(s^2+4)}$$

2. (20 points) (RC synthesis) Replace each L by an R in the above syntheses and give the resulting RC admittances. Compare degrees of the different y(s) obtained, including LC versus RC.

3. (25 points) (Minimum y(s)) Show that $y(s)=[s^2+s+2]/\{s^2+s+1\}$is PR with no pole or zero on the jω axis. Sketch Re(y(jω) and find the minimum value of Re(y(jω))=g. Show that z1(s)=1/[y(s)-g] is PR but still without a pole on the jω axis; locate the even part zeroes of z1(s) and y(s) and compare.

3. (35 points) (Synthesis from state equations) Set up the companion matrix form of state-variable equations for the admittance $y(s) = \frac{3s^2+2s+1}{s^2+3}$. Check if this is a PR y(s). Synthesize the resulting 3-port admittance matrix, loaded in two unit capacitors, and check by analyzing the circuit that y(s) results.

(Research type problem). Show that the following is positive-real and discuss the position and nature of its singularities (note that a limit of poles is an essential singularity and all real numbers are limits of rational numbers).

$$y(s) = \sum_{k=1}^{\infty}\sum_{m=1}^{\infty}\coth(s\tfrac{k}{m})$$

Related to this is that the following formulas hold for the lossless positive-real tanh and cotanh=coth which are related to transmission lines.

$$\tanh(s) = 2s[\sum_{n=0}^{\infty}\frac{1}{(s^2+(n+\frac{1}{2})^2\pi^2)}]$$

$$\coth(s) = \frac{1}{s} + 2s[\sum_{n=1}^{\infty}\frac{1}{(s^2+(n\pi)^2)}]$$