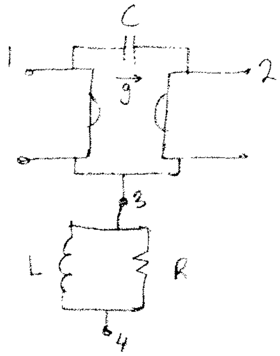
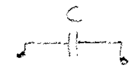
Homework # 4

Problem # 1

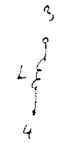


$$i_1 = g(V_2 - V_3)$$
$$i_3 = g(V_3 - V_2)$$

$$i_2 = g(V_3 - V_1)$$
$$i_3 = g(V_1 - V_3)$$



$$i_1 = sC(V_1 - V_2)$$
$$i_2 = sC(V_2 - V_1)$$



$$i_3 = \frac{1}{sL}(V_3 - V_4)$$
$$i_4 = \frac{1}{sL}(V_4 - V_3)$$

$$i_3 = G_R(V_3 - V_4)$$
$$i_4 = G_R(V_4 - V_3)$$

$$Y_{ind} = \begin{bmatrix} sC & -sC+g & -g & 0 \\ -sC-g & sC & g & 0 \\ g & -g & \frac{1}{sL}+G_R & -\frac{1}{sL}-G_R \\ 0 & 0 & -\frac{1}{sL}-G_R & \frac{1}{sL}+G_R \end{bmatrix}$$

Ground node 4:

$$Y_{3\times 3} = \begin{bmatrix} sC & -sC+g & -g \\ -sC-g & sC & g \\ g & -g & \frac{1}{sL}+G_R \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}$$

$$Y_{2port} = Y_{11} - Y_{12} Y_{21} Y_{22}^{-1}$$

$$= \begin{bmatrix} sC & -sC+g \\ -sC-g & sC \end{bmatrix} - \frac{1}{1/sL + G_R} \begin{bmatrix} -g \\ g \end{bmatrix} \begin{bmatrix} g & -g \end{bmatrix}$$

$$= \begin{bmatrix} sC & -sC+g \\ -sC-g & sC \end{bmatrix} - \frac{1}{1/sL + G_R} \begin{bmatrix} -g^2 & g^2 \\ g^2 & -g^2 \end{bmatrix}$$

$$Y_{2port} = \begin{bmatrix} sC + \dfrac{sLg^2}{1+sLG_R} & g - sC - \dfrac{sLg^2}{1+sLG_R} \\ -g - sC - \dfrac{sLg^2}{1+sLG_R} & sC + \dfrac{sLg^2}{1+sLG_R} \end{bmatrix} = Y(s)$$

$$Z(s) = Y^{-1}(s)$$

$$Z(s) = \begin{bmatrix} \dfrac{sC}{g^2} + \dfrac{sL}{(1+sLG_R)} & \dfrac{-1}{g} + \dfrac{sC}{g^2} + \dfrac{sL}{(1+sLG_R)} \\[2ex] \dfrac{1}{g} + \dfrac{sC}{g^2} + \dfrac{sL}{(1+sLG_R)} & \dfrac{sC}{g^2} + \dfrac{sL}{(1+sLG_R)} \end{bmatrix}$$

Problem #2

· Even Part of matrix $A(s) = \frac{1}{2}(A(s) + A(-s))$

$\therefore$ Even Part of $Y(s)$ $= \frac{1}{2}\left(Y(s) + Y(-s)\right)$

$$= \frac{1}{2}\left[\begin{bmatrix} sC + \dfrac{sLg^2}{1+sLG_R} & g - sC - \dfrac{sLg^2}{1+sLG_R} \\[2ex] -g - sC - \dfrac{sLg^2}{1+sLG_R} & sC + \dfrac{sLg^2}{1+sLG_R} \end{bmatrix} + \begin{bmatrix} -sC - \dfrac{sLg^2}{1-sLG_R} & g + sC + \dfrac{sLg^2}{1-sLG_R} \\[2ex] -g + sC + \dfrac{sLg^2}{1-sLG_R} & -sC - \dfrac{sLg^2}{1-sLG_R} \end{bmatrix}\right]$$

$$\text{Even part of } Y(s) = \begin{bmatrix} \dfrac{s^2L^2g^2G_R}{s^2L^2G_R^2 - 1} & g - \dfrac{s^2L^2g^2G_R}{s^2L^2G_R^2 - 1} \\[2ex] -g - \dfrac{s^2L^2g^2G_R}{s^2L^2G_R^2 - 1} & \dfrac{s^2L^2g^2G_R}{s^2L^2G_R^2 - 1} \end{bmatrix}$$

Similarly:

$$\text{Even part of } Z(s) = \begin{bmatrix} \dfrac{s^2L^2G_R}{s^2L^2G_R^2 - 1} & \dfrac{s^2L^2G_R^2}{s^2L^2G_R^2 - 1} - \dfrac{1}{g} \\[2ex] \dfrac{s^2L^2G_R^2}{s^2L^2G_R^2 - 1} + \dfrac{1}{g} & \dfrac{s^2L^2G_R^2}{s^2L^2G_R^2 - 1} \end{bmatrix}$$

Zeroes

$Y(s)$

$$g - \frac{s^2L^2g^2G_R}{s^2L^2G_R^2 - 1} = 0 \qquad\qquad -g - \frac{s^2L^2g^2G_R}{s^2L^2G_R^2 - 1} = 0$$

$$gs^2L^2G_R^2 - g = s^2L^2g^2G_R \qquad\qquad -gs^2L^2G_R^2 + g = s^2L^2g^2G_R$$

$$s = \pm\frac{1}{L}\sqrt{\frac{1}{G(G-g)}} \qquad\qquad s = \pm\frac{1}{L}\sqrt{\frac{1}{G(G+g)}} \qquad\qquad s = 0$$

$\therefore$ Zeros of Eq $Y(s)$ $\Rightarrow$ $s = 0, \pm\sqrt{\frac{1}{G(G\pm g)}}$

$Z(s)$

$$\frac{s^2L^2G_R^2}{s^2L^2G_R^2 - 1} - \frac{1}{g} = 0 \qquad\qquad \frac{s^2L^2G_R^2}{s^2L^2G_R^2 - 1} - \frac{1}{g} = 0 \qquad\qquad s = 0$$

$$s = \pm\frac{1}{L}\sqrt{\frac{1}{G(G-g)}} \qquad\qquad s = \pm\frac{1}{L}\sqrt{\frac{1}{G(G+g)}}$$

Zeros of Eq $Z(s)$ $\Rightarrow$ $s = 0, \pm\frac{1}{L}\sqrt{\frac{1}{G(G\pm g)}}$

- [illegible] proportional to G_R

Problem #3

With $C = G = 0$ we obtain:

$$Y(s) = \begin{bmatrix} sLg^2 & g - sLg^2 \\ -g - sLg^2 & sLg^2 \end{bmatrix} \qquad Z(s) = \begin{bmatrix} sL & sL - \frac{1}{g} \\ sL + \frac{1}{g} & sL \end{bmatrix} \qquad \Delta Z = \frac{1}{g^2}$$

Find $Z_L(s)$ in terms of $Z_{in}(s)$

$$\underbrace{\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} \qquad \begin{matrix} i_2 = -\frac{1}{Z_L} v_2 \\ v_2 = -Z_L i_2 \end{matrix}}$$

$$v_2 = Z_{21} i_1 + Z_{22} i_2 \qquad \text{(row 2)}$$

$$-Z_L i_2 = Z_{21} i_1 + Z_{22} i_2$$

$$i_2 = (-Z_{21})(Z_{22} + Z_L)^{-1} i_1$$

$$v_1 = Z_{11} i_1 + Z_{12} i_2$$

$$v_1 = Z_{11} i_1 - Z_{12} Z_{21} (Z_{22} + Z_L)^{-1} i_1$$

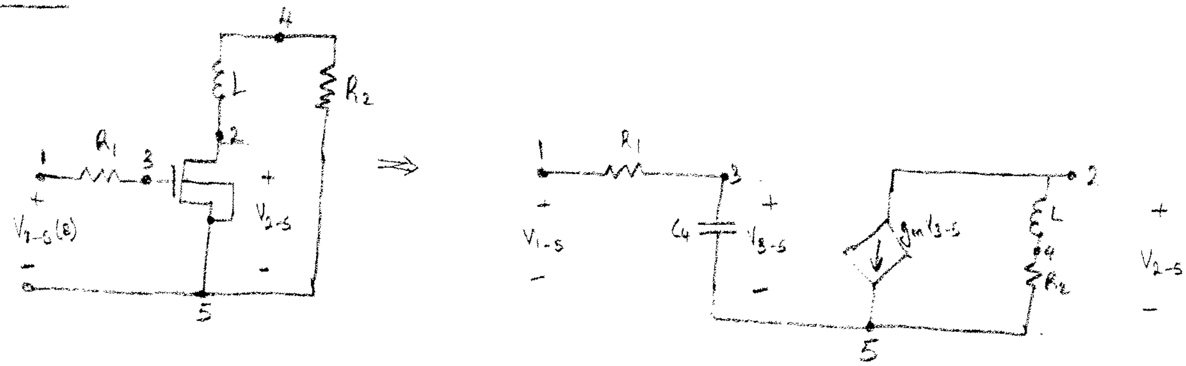
$$\frac{v_1}{i_1} = Z_{in} = Z_{11} - \frac{Z_{12} Z_{21}}{Z_{22} + Z_L} \longrightarrow Z_L = \frac{Z_{in} Z_{22} - [Z_{11} Z_{22} - Z_{12} Z_{21}]}{Z_{11} - Z_{in}}$$

$$Z_L = \frac{Z_{in} Z_{22} - \Delta Z}{Z_{11} - Z_{in}}$$

Plugging in for $Z(s)$

$$Z_L = \frac{Z_{in} sL - \frac{1}{g^2}}{sL - Z_{in}}$$

Problem 4



· find Y-2port:

R_1

$i_1 = G_1(V_1 - V_3)$

$i_3 = G_1(V_3 - V_1)$

C_4

$i_3 = sC_4(V_3 - V_5)$

$i_5 = sC_4(V_5 - V_3)$

g_m

$i_2 = g_m(V_3 - V_5)$

$i_5 = g_m(V_5 - V_3)$

L

$i_2 = \frac{1}{sL}(V_2 - V_4)$

$i_4 = \frac{1}{sL}(V_4 - V_2)$

R_2

$i_4 = G_2(V_4 - V_5)$

$i_5 = G_2(V_5 - V_4)$

→ Get indefinite admittance:

$$Y_{indef} = \begin{bmatrix} G_1 & 0 & -G_1 & 0 & 0 \\ 0 & 1/sL & g_m & -1/sL & -g_m \\ -G_1 & 0 & G_1 + sC_4 & 0 & -sC_4 \\ 0 & -\frac{1}{sL} & 0 & G_2 + \frac{1}{sL} & -G_2 \\ 0 & 0 & -g_m - sC_4 & -G_2 & g_m + sC_4 + G_2 \end{bmatrix}$$

→ Input / output will be in terms of V_5 so let's take it as common ground → Y_{4x4}

→ Now eliminate nodes 3 and 4 → $Y_{2\text{-port}}$

$$Y_{4x4} = \begin{bmatrix} G_1 & 0 & -G_1 & 0 \\ 0 & 1/sL & g_m & -1/sL \\ -G_1 & 0 & G_1 + sC_4 & 0 \\ 0 & -1/sL & 0 & G_2 + \frac{1}{sL} \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}$$

$$Y_{2\text{-port}} = Y_{11} - Y_{12} Y_{22}^{-1} Y_{21}$$

$$Y_{2\text{-port}} = \begin{bmatrix} \dfrac{G_1 sC_4}{G_1 + sC_4} & 0 \\ \dfrac{g_m G_1}{G_1 + sC_4} & \dfrac{G_2}{sLG_2 + 1} \end{bmatrix}$$

$\therefore$

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = Y_{s\text{-}pxt} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

with open-circuit load, $i_2 \to 0$

$\therefore$ from row #2:

$$0 = \frac{g_m G_1}{G_1 + sC_4} V_1 + \frac{G_2}{sLG_2 + 1} V_2$$

$\rightarrow$ solve for V_2/V_1:

$$\frac{V_2}{V_1}(s) = \frac{-g_m G_1 (sLG_2 + 1)}{G_2 (G_1 + sC_4)} = \frac{-g_m (sL + 1/G_2)}{1 + sC_4/G_1}$$

$$\therefore \quad T(s) = \frac{-g_m (R_2 + sL)}{1 + sC_4 R_1}$$

Compare to result from KCL/KVL on small signal

$$V_{gs} = V_1 \frac{1/sC_4}{1/sC_4 + R_1} \qquad -g_m V_{gs} = \frac{V_2}{R_2 + sL}$$

$$\Rightarrow \quad -g_m \frac{1}{sC_4R_1 + 1} V_1 = V_2 \frac{1}{R_2 + sL} \longrightarrow \frac{V_2}{V_1} = \frac{-g_m (R_2 + sL)}{1 + sC_4 R_1}$$

b) $C_4 = C \quad R_1 = R_2 = R \quad R = C = 1$

$$\therefore \quad T(s) = \frac{-g_m (1 + sL)}{1 + s}$$

poles: $s = -1$

zeros: $s = -1/L$

c) Step Response: $V_{1-S}(s) = \frac{1}{S}$

$$\therefore V_{2-S}(s) = \frac{1}{S} \cdot \frac{-g_m(1+sL)}{1+S} = \frac{-g_m}{S} + \frac{g_m(1-L)}{1+S}$$

$$\mathcal{L}^{-1}\left[\frac{-g_m}{S} + \frac{1-L}{1+S}\right] = \left[-g_m + g_m(1-L)e^{-t}\right]u(t)$$

$$\therefore \quad V_{2-S}(t) = \left[-g_m + g_m(1-L)e^{-t}\right]u(t)$$

Impulse Response

$$\frac{d}{dt}V_{2-S}(t) = -g_m L\,\delta(t) - g_m(1-L)e^{-t}u(t)$$