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610 Fall 2013 – Homework 2 due Th 09/26/13

1. (50 points, semistate equations)

A circuit has the following semistate matrices, for Edx/dt=Ax+Bu, y=Cx,

$$E := \begin{pmatrix} 0 & 0 & 0 & 0 \\ C1 & 0 & 0 & 0 \\ 0 & 0 & 0 & L1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad A := \begin{pmatrix} 0 & g1 & 0 & 0 \\ -g1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ g1 & 0 & -1 & 0 \end{pmatrix} \quad B := \begin{pmatrix} 0 & 1 \\ 0 & 1 \\ 1 & 0 \\ 1 & 0 \end{pmatrix} \quad C := \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Assuming that C1, L1 and g1 are all positive

a) Using permutations transform the system so that E is the direct sum of 1_2 and 0_2.

b) Eliminate the last two rows (by solving for the last two components of x in terms of the first two components) to obtain state variable equations.

b) Find the transfer function 2x2 matrix, $T(s)=C(sE-A)^{-1}B$, where E is the above 4x4 matrix. Check by using the state variable equations (which have a Du term in the output, y, equation).

Additional Problem Not for Grading (RC phase shift oscillator)

The following circuit is an RC phase shift oscillator. .

a) Replace the npn transistor by a small signal pi equivalent circuit (using only r_π and g_m assuming go=0 and ignoring C_π). Find a set of semistate equations when C1=C2=C3=c, R1=R2=RL=1/g, $r_\pi = \beta/g_m$ (ignore the biasing components Ra, Rb, RE, Cbypass). Use branches of C1, C2, C3 and RL for a tree and include series initial capacitor voltages, ICs, as terms in the input u (via unit step functions), orienting them + on the left side. Take vL as the output.

b) Find the characteristic polynomial, p(s)=det(Es-A), and from it show that c/g and bias collector current I_C for $gm=I_C/V_T$, $g_\pi=g_m/\beta$ can be used to force $p(j\omega_o)=0$ for some real ω_o. (the Spice model for the 2N2222 has β=256). Determine the range of ω_o for which c/g is positive (giving a realizable oscillator)