EE 303 H
10/17/13

Note Title 10/17/2013

$$T(s) = \frac{1}{s^2 + a_{n-1}s^{n-1} + \cdots a_1 s + 1}$$ on normalization

$$\frac{1}{|T(j\omega)|^2} = \frac{1}{T(j\omega)T^*(j\omega)} = \frac{1}{T(j\omega)\,T(-j\omega)} = 1 + \omega^{2n}$$

$$\Rightarrow \frac{1}{T(s)}\cdot\frac{1}{T(-s)} = 1 + (-1)^n s^{2n} \Rightarrow P(s) = 1 + (-1)^n s^{2n}$$

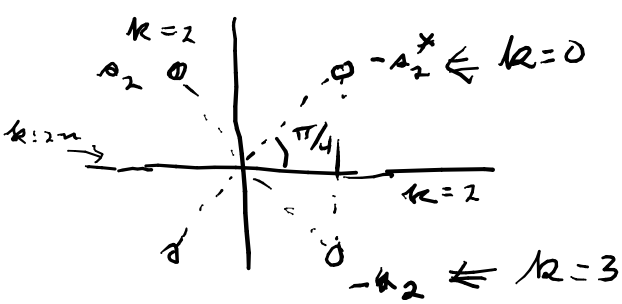
desire its zeros

$$T(s)T(-s) = P(s) = 1 + (-1)^n s^{2n} \Rightarrow s^{2n} = (-1)^{+n+1} = \begin{cases} -1 & n \text{ even} \\ + & n \text{ odd} \end{cases}$$

n even $\quad s^{2n} = -1$; $\quad s_{root} = 2n$th roots of -1; $\quad -1 = e^{j(\pi + 2\pi k)}$

$$s_k = e^{j\frac{(\pi + 2\pi k)}{2n}}\ ; \quad k = 0, 1, \ldots, 2n$$

$n = 2$



$\underbrace{\qquad}_{T(s)}$ $\quad \underbrace{\qquad}_{T(-s)}$

$n = 2$, $k = 1$ $\angle = 3\pi/4$

$= 2$ $\angle = 5\pi/4$

$= 0$ $\angle = \pi/4$

$k = 3$ $\angle = 7\pi/4$

$$s_2 = e^{j\,5\pi/4}$$
$$= -\cos 45^\circ + j\sin 45^\circ$$
$$= -\frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}$$

$n = 2$

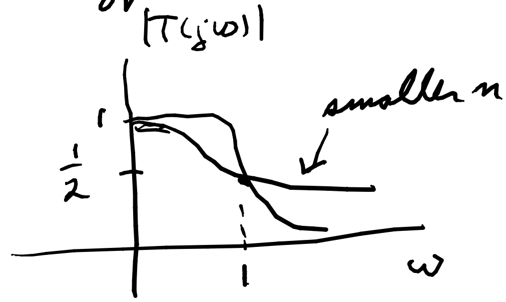
$$T(s) = \frac{1}{s^2 + a_1 s + 1}$$
$$= \frac{1}{(s - s_2)(s - s_2^*)}$$
$$= \frac{1}{\left(s + \frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}\right)\left(s + \frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}\right)}$$
$$= \frac{1}{s^2 + \frac{2}{\sqrt{2}}s + 1}$$

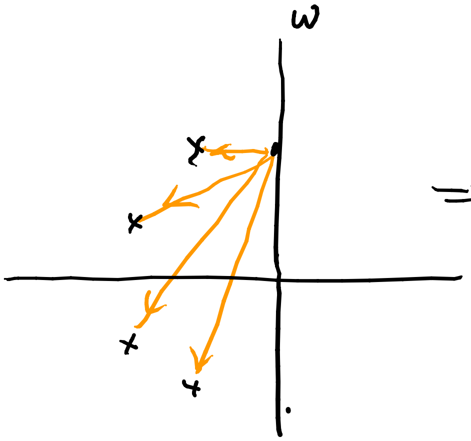
$T(s) = \dfrac{1}{s^2 + \sqrt{2}\,s + 1}$ for any n $\quad 1 + (-1)^n s^{2n}$

@ $s = j1$ $\quad = 1 + (-1)^n (j)^{2n} = 2$

$\therefore$ @ $j\omega = j1$, $|T(s)| = \dfrac{1}{2}$ = cut off

$j\omega$
max flat

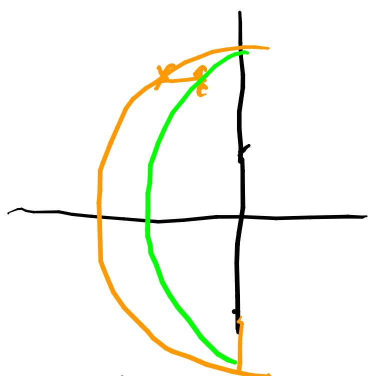


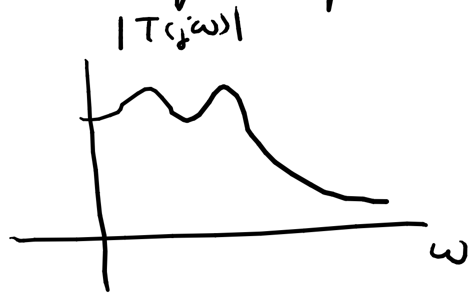


$\Rightarrow$ $T(s) = \dfrac{1}{\Sigma \text{ mag}(j\omega \text{ to pole term})}$

for poles on unit circle & equally spaced in & get max flat

So if poles onto an ellipse can get equal ripple





Given $T(s) = \dfrac{c_{n-1}s^{n-1} + \dots + c_0}{s^n + a_{n-1}s^{n} + \dots + a_1 s + a_0} = \dfrac{y}{u}$

how to design a circuit?

$$\begin{bmatrix} \dot{x} \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \cdots & 1 \\ 0 & 0 & & & & \\ -a_0 & -a_1 & \cdots & & & -a_{n-1} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ b \end{bmatrix} u$$

companion matrix form

$$y = \begin{bmatrix} c_0 & \cdots & c_{n-1} \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}$$

$\bar{x} = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}$ = state

$$\dot x_1 = x_2 = \quad x_2 = s x_1 \qquad s = d/dt$$

$$\dot x_2 = x_3 = \ddot x_1, \quad x_3 = s^2 x_1$$

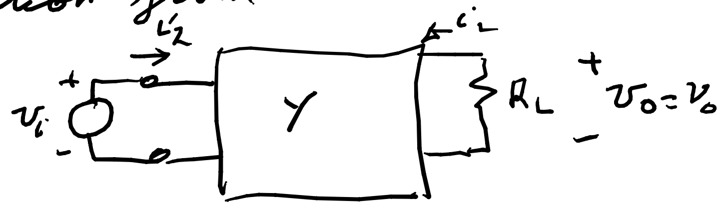
$$s x_n = s^n x_1 = -a_0 x_1 - a_1 s x_1 - \cdots - a_{n-1} s^{n-1} x_1 + bu$$

$$(s^n + a_{n-1} s^{n-1} + \cdots + a_0) x_1 = bu$$

$$y = [c_0 \cdots \quad c_{n-1}] \begin{bmatrix} x_1 \\ s x_1 \\ \vdots \\ s^{n-1} x_1 \end{bmatrix}$$

$$= [c_{n-1} s^{n-1} + \cdots c_0] x_1 = \frac{[c_{n-1} s^{n-1} + \cdots + c_0] b u}{s^n + a_{n-1} s^{n-1} + \cdots + a_0}$$

To create a transfer voltage function from admittances



$\dfrac{v_0}{v_i}$

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}; \quad i_2 = -G_L v_0$$

$$= y_{21} v_1 + y_{22} \underset{v_2}{\underset{\parallel}{v_0}}$$

$$(-G_L - y_{22}) v_0 = y_{21} v_i$$

$$\frac{v_0}{v_i} = \frac{-y_{21}}{G_L + y_{22}} \qquad \left(ex = \frac{1}{s^2 + \sqrt{2} s + 1} \right)$$

$$\text{if } G_L = 1 \Rightarrow \quad \frac{v_0}{v_i} = \frac{1/(s^2+1)}{1 + \frac{\sqrt{2} s}{s^2+1}}$$