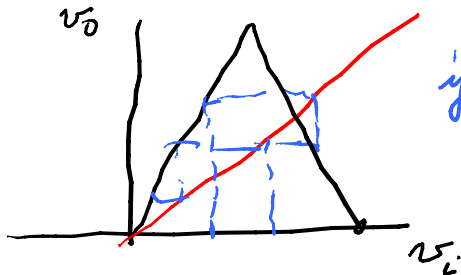
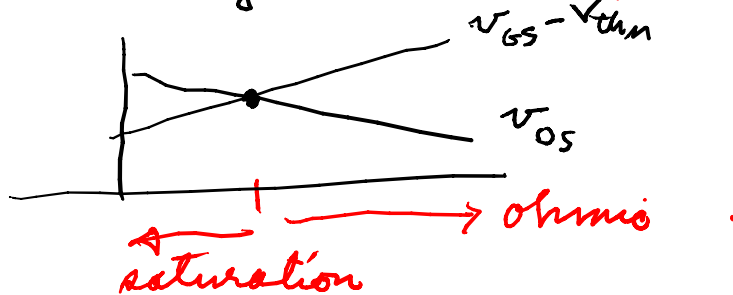


For chaos look at a return map



if an initial value returns to itself after 3 returns then this will give "chaos"

To find state of an NMOS *from model*



For v_{iH} , M_n is ohmic & M_p is saturated

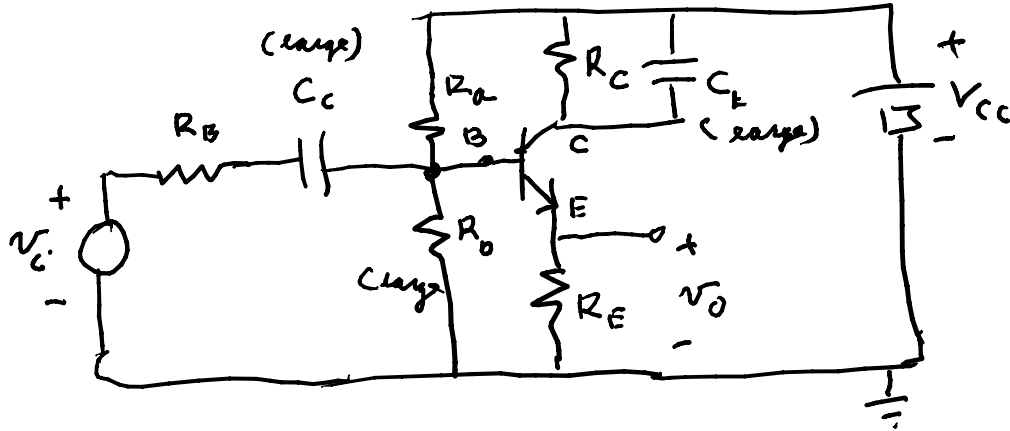
$$\begin{aligned}
 i_{Sp} = i_{Dn} &= k_p (V_{DD} - v_i - |V_{T0,p}|)^2 (1 + \lambda_p (V_{DD} - v_o)) \\
 &= k_n (2(v_i - V_{T0,n})v_o - v_o^2) (1 + \lambda_n v_o)
 \end{aligned}
 \left. \vphantom{\begin{aligned} i_{Sp} = i_{Dn} \\ = k_n \dots \end{aligned}} \right\} \begin{array}{l} \text{difference} \\ = f(v_i, v_o) \end{array}$$

for slope = -1 = $\frac{dv_o}{dv_i}$

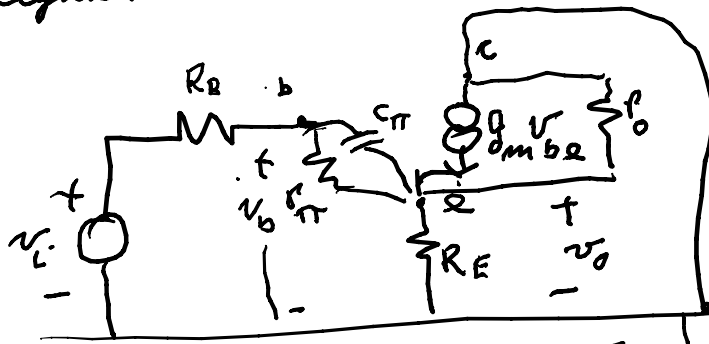
But note $F(v_i, v_o(v_i)) + 1 = 0$

$$\Rightarrow \frac{dv_o}{dv_i} = - \frac{\partial F(v_i, v_o(v_i)) / \partial v_i}{\partial F(v_i, v_o(v_i)) / \partial v_o} = -1$$

Solve for $v_o(v_i)$, a quadratic eq. in v_o & substitute into $F(v_i, v_o(v_i))$ to find v_i .



For small signal



$$\frac{g_m}{\beta_F} = g_{\pi} = \frac{1}{r_{\pi}} \quad g_m = \frac{I_C}{V_T}, \quad \beta_0 = \frac{I_C}{I_{AF}} \quad V_T \approx 0.26 \text{ mV}$$

derive $v_o/v_i(A)$ use KCL, sum currents into a node to 0

@ b: $G_B(v_i - v_b) - (g_{\pi} + \beta_0)(v_b - v_e) = 0 = G_B(v_i - v_o) - g_{\pi}(v_o - v_e)$

@ e: $-G_E v_o + g_m(v_b - v_e) - g_o v_o + g_{\pi}(v_o - v_e) = 0$

$$\begin{bmatrix} G_B + g_{\pi} & -g_{\pi} \\ -(g_m + g_{\pi}) & G_E + g_m + g_o + g_{\pi} \end{bmatrix} \begin{bmatrix} v_b \\ v_e \end{bmatrix} = \begin{bmatrix} G_B v_i \\ 0 \end{bmatrix} \quad \text{here } v_e = v_o$$

$$v_b = \frac{(G_E + g_m + g_o + g_{\pi})}{g_m + g_{\pi}} \cdot v_o \Rightarrow 2^{\text{nd}} \text{ eq.}$$

$$\Rightarrow \left[\frac{G_B + Y_{\pi}}{g_m + Y_{\pi}} (G_E + g_m + g_o + Y_{\pi}) - Y_{\pi} \right] v_o = G_B v_i$$

$$\frac{v_o}{v_i}(s) = \frac{G_B (g_m + Y_{\pi} + R_C Y_{\pi})}{(G_B + g_m + R_C Y_{\pi})(G_E + g_m + g_o)} = \frac{a + b}{c + d}$$

$\begin{matrix} G_B(g_m + Y_{\pi}) & G_B C_{\pi} \\ \parallel & \parallel \\ a + b & \\ \parallel & \parallel \\ (G_E + g_m + g_o) C_{\pi} & \end{matrix}$

for unit step response, use $v_i = \frac{1}{s}$

$$v_o(s) = \frac{a + b}{c + d} \times \frac{1}{s} = \frac{K_1}{s} + \frac{K_2}{s + c/d} = \frac{a}{c} + \frac{a - b(c/d)}{-c/d} \frac{1}{s + c/d}$$

$v_i = 1(s)$

$$v_o(t) = \frac{a}{c} 1(t) + \left(\frac{a - b(c/d)}{-c/d} \right) e^{-(c/d)t} 1(t)$$

Impulse response. $\mathcal{L}^{-1} \left(\frac{v_o(s)}{v_i = 1(s)} \right) / dt$

$$v_o(t) \Big|_{s=v_i} = \frac{a}{c} \delta(t) + (a - b(c/d)) e^{-(c/d)t} 1(t) + \left(\frac{a - b(c/d)}{-c/d} \right) \delta(t)$$

Frequency response

$$\text{mag.} \left(\frac{v_o}{v_i}(s) \right) \Big|_{s=j\omega} = \left| \frac{a + j\omega b}{c + j\omega d} \right| = \frac{|a + j\omega b|}{|c + j\omega d|}$$