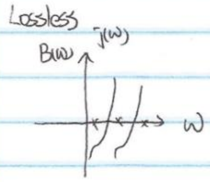
EE 610 10/04/12

Lossless

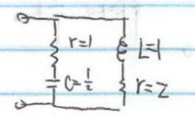


always a zero or a pole at $s=0$ & $s=\infty$

$y(s) = -y(-s) = \frac{1}{s} Ev(s)$ $v = s\,Ev(s)$ $a_0 s \frac{s^2+\omega_1^2}{s^2+\omega_{p1}^2}$ $a_0 > 0$

Look at

$y(s) = \frac{s+1}{s+2} = \frac{s+1}{s+2} = \frac{1}{1+\frac{2}{s}} + \frac{1}{s+2}$



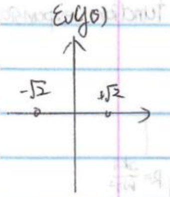
$\delta[y(s)] = 1$, but uses 2 reactive elements.

$\therefore$ uses the Richard Function

$y_L(s) = y(k)\,\frac{k\,y(k) - s\,y(s)}{k\,y(s) - s\,y(k)}$ $s-k$ is a factor

If k is a zero of $Ev(y(s))$ then $s+k$ also cancel

(i.e. $s-k$ gives $0/0$ & $s+k$ cancels)

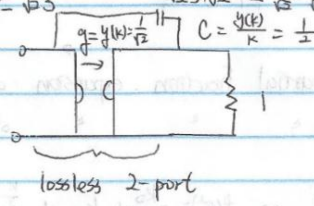


$E_v[y(s)] = \frac{1}{2}\left(y(s) + y(-s)\right) = \frac{1}{2}\left[\frac{s+1}{s+2} + \frac{-s+1}{-s+2}\right] = \frac{-s^2+2}{-s^2+4}$

$E_v\,y(s) = 0$ $-s^2+2=0$ $s^2=2$, $s=\pm\sqrt{2}$ $\Rightarrow$ $k=\sqrt{2}$, $y(k) = \frac{\sqrt{2}+1}{\sqrt{2}+2} = \frac{1}{\sqrt{2}}\frac{1+\sqrt{2}}{(\sqrt{2}+1)} = \frac{1}{\sqrt{2}}$

$y_L(s) = \frac{1}{\sqrt{2}}\,\frac{\sqrt{2}\cdot\frac{1}{\sqrt{2}} - s\frac{s+1}{s+2}}{\sqrt{2}\frac{s+1}{s+2} - \frac{1}{\sqrt{2}}s} = \frac{1}{\sqrt{2}}\,\frac{s+2-s^2-s}{\sqrt{2}s+\sqrt{2} - \frac{s^2}{\sqrt{2}} - \frac{2s}{\sqrt{2}}} = \frac{1}{\sqrt{2}}\,\frac{2-s^2}{\frac{2-s^2}{\sqrt{2}}} = 1$

$y(s) = \frac{s+1}{s+2} \longrightarrow$

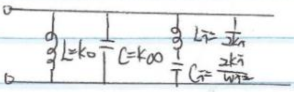


$y_{LC}(s) = s\,E_v\,f(s) = s\,f(s^2)$

y_{LC}; 2nd Froster

$y_{LC} = \frac{k_0}{s} + k_\infty s + \sum_{i=1}^{K} \frac{2k_i s}{s^2+\omega_i^2} = \frac{k_0}{s} + k_\infty s + \sum_{i=1}^{k} \frac{2k_i}{s+\frac{\omega_i^2}{s}}$

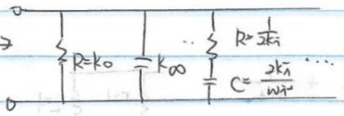
$y_{LC} \longrightarrow$



$\Downarrow$

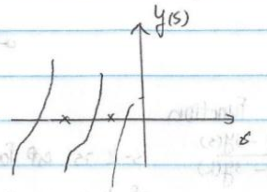
replaces L's by R's

$y_{RC}(s) \longrightarrow$



$$y_{RC}(s) = k_0 + k_\infty s + \sum_{i=1}^{k} \frac{1}{\left(\frac{1}{2k_i} + \frac{w_i^2}{2k_i s}\right)}$$

$$= k_0 + k_\infty s + \sum_{i=1}^{k} \frac{2k_i s}{s + w_i^2}$$



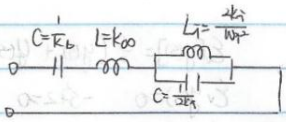
all poles are on $-\sigma$ axis, $\frac{dy(\sigma)}{d\sigma}$:

$$\frac{y_{RC}(s)}{s} = \frac{k_0}{s} + k_\infty + \sum_{i=1}^{k} \frac{2k_i}{s + w_i^2}$$

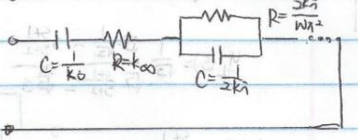
is a partial function expansio

1st Foster

$$z_{LC}(s) = \frac{k_0}{s} + k_\infty s + \sum_{i=1}^{k} \frac{2k_i s}{s^2 + w_i^2}$$



$$z_{RC}(s) = \frac{k_0}{s} + k_\infty + \sum_{i=1}^{k} \frac{2k_i}{s + w_i^2}$$



1st Foster $\Rightarrow$ partial fraction expansion of $z_{RC}(s)$

2nd " $\Rightarrow$ " " " of $\frac{y_{RC}(s)}{s}$

$y_{LC}(s) = s f(s^2) \Rightarrow f(s^2) = \frac{k_0}{s^2} + k_\infty + \sum_{i=1}^{k} \frac{2k_i}{s^2 + w_i^2}$

$\frac{y_{RC}(s)}{s} = f(s)$

1st Cauer's $\Rightarrow$ remove a pole at ∞ of $y_{RC}(s)$ if it exists
otherwise remove the constant at ∞ of $z_{RC}(s) = \frac{1}{y_{RC}(s)}$

2nd Cauer's $\Rightarrow$ remove pole of $z_{RC}(s)$ at 0 and constant of $y_{RC}(s)$ at 0

$$y(s) = \cancel{\frac{b_0}{s+a_0}} \quad \left(\frac{i}{v}\right) \quad \text{companion matrix form for state variable}$$

$$\frac{b_1 s + b_0}{s^2 + a_1 s + a_0}$$

$$\frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -a_0 & -a_1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} v \qquad i = [C_1 \; C_2]\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

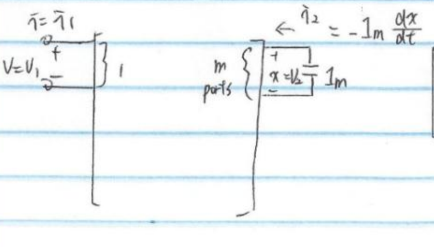
$$\frac{v}{i} = [C_1 \; C_2]\begin{bmatrix} s & -1 \\ a_0 & s+a_1 \end{bmatrix}^{-1}\begin{bmatrix} 0 \\ 1 \end{bmatrix} \qquad \begin{bmatrix} s & -1 \\ a_0 & s+a_1 \end{bmatrix}^{-1} = \frac{1}{s^2+a_1 s+a_0}\begin{bmatrix} s+a_1 & 1 \\ -a_0 & s \end{bmatrix}$$

$$= \frac{[C_1, C_2]}{s^2+a_1 s+a_0}\begin{bmatrix} a_1+s & 1 \\ -a_1 & s \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \frac{C_1 + C_2 s}{s^2+a_1 s+a_0} \qquad \text{Let } C_1 = b_0, \; C_2 = b_1$$

$$\text{For } y(s) = \frac{\sum_{i=0}^{m-1} b_i s^i}{\sum_{j=0}^{m} a_j s^j} \Rightarrow \frac{d}{dt}\begin{bmatrix} x_1 \\ \vdots \\ x_m \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & \\ 0 & & & \ddots & \\ \vdots & & & & 1 \\ -a_0 & -a_1 & \cdots & & -a_{m-1} \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix} v$$

$-i_2 =$

$$i = [b_0 \; b_1 \cdots b_{m-1}]\, x$$



$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \underbrace{\left[\begin{array}{c|ccccc} 0 & C_1 & \cdots & & & C_m \\ \hline 0 & 0 & -1 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & & & \ddots & \\ -1 & \vdots & & & & 1 \\ & a_0 & a_1 & \cdots & & a_{m-1} \end{array}\right]}_{Y_c} \begin{bmatrix} V_1 \\ V_2 = X \end{bmatrix}$$