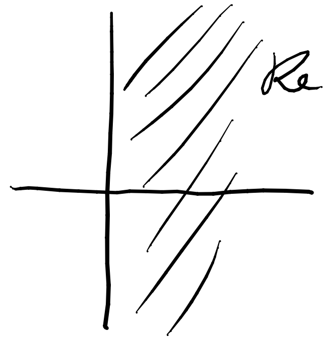
EE 610
12/10/12

Note Title 12/11/2012

PR $\Rightarrow$ no poles in $\sigma > 0$

coefficients are ≥ 0

$\mathrm{Re}\, y(j\omega) \geq 0$

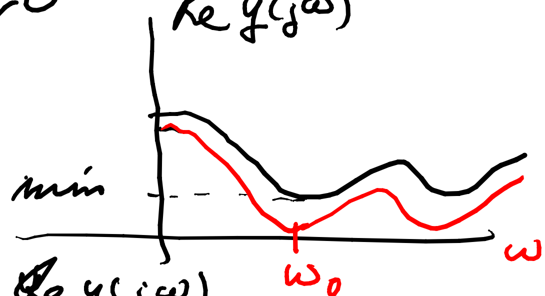


$e^{\pm y(s)}$ by max modulus theorem $|e^{\pm y(s)}|$ max on a boundary where $y(s)$ is analytic

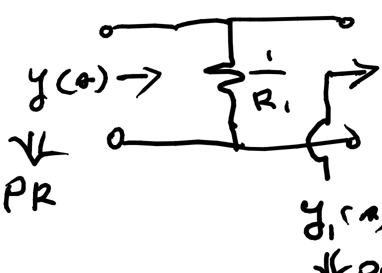
$\Rightarrow$ for a PR $y(s) \Rightarrow |e^{\pm y(j\omega)}|$ is max of $|e^{-y(s)}|$ for s in the RHP

$= e^{\mp \mathrm{Re}\, y(j\omega)}$

$\Rightarrow y(s)$ PR $\Rightarrow \mathrm{Re}\, y(j\omega) \geq 0$



$y_1(s) = y(s) - R_1$ is still PR, $\quad R_1 = \min\limits_{\omega \geq 0} \mathrm{Re}\, y(j\omega)$



$\Rightarrow \mathrm{Re}\, y_1(j\omega_0) = 0$

Bott-Duffin

p.360 $\Rightarrow$ transformerless synthesis of a PR function by positive R, L, C

$$R(s) = \frac{k Z(s) - s Z(k)}{k Z(k) - s Z(s)}, \quad Z(s) \Rightarrow \text{PR}$$

$$\Rightarrow Z(s) = \frac{k Z(k) R(s) + s Z(k)}{k + s R(s)} = \frac{k Z(k) R(s)}{k + s R(s)} + \frac{s Z(k)}{k + s R(s)}$$

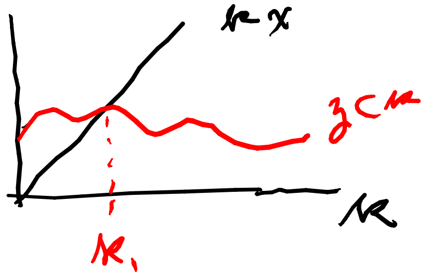
assume $\underbrace{z(j\omega_0) = jx}$ $= \dfrac{k\, z(k) R(j\omega_0) + j\omega_0 z(k)}{k + j\omega_0 R(j\omega_0)}$

$$R(j\omega_0) = \frac{k\, z(j\omega_0) - j\omega_0 z(k)}{k\, z(k) - j\omega_0 \cdot j z(j\omega_0)} = \frac{j(kx - \omega_0 z(k))}{k\, z(k) + \omega_0 x}$$

if $k>0$ then $\dfrac{R(s)}{z(k)}$ is PR & we will create a zero or a pole at $s = j\omega_0$

if $x>0$ then find $k>0$ such that $R(j\omega_0) = 0$

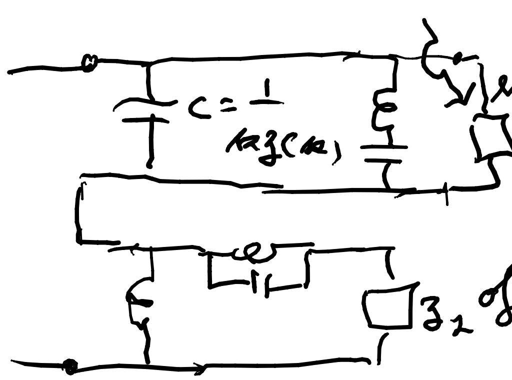
$$kx = \omega_0 z(k) \qquad x = \frac{1}{j} z(j\omega_0) > 0$$



using the solution $k = k_1 > 0$ gives $R(s)$, PR with a zero at $s = j\omega_0$ of both real and imaginary parts

$$z(s) = \frac{k\, z(k) R(s)}{k + sR(s)} + \frac{s\, z(k)}{k + sR(s)} = \frac{1}{\frac{1}{z(k)R(s)} + \frac{s}{k\, z(k)}} + \frac{1}{\frac{1}{\frac{s\, z(k)}{k}} + \frac{R(s)}{z(k)}}$$

this is the sum of 2 PR functions & the $R(s)$ terms have $R(j\omega_0) = 0$



extracted pole at $s = j\omega_0$

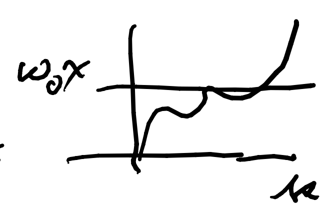
$z_1(s)$ of smaller degree & PR

$z(s)$

z_2 of smaller degree & PR

continue on z_1 & z_2

passive PR terms } transformerless



also works for $x<0$ by using $k\, z(k) = -\omega_0 x$

Brune synthesis of PR functions p. 357

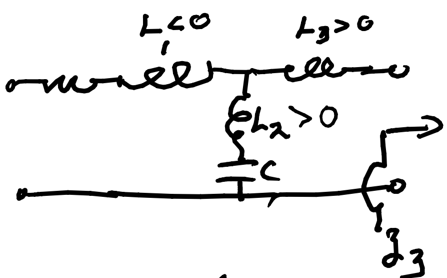
$z(s) - R_1 \Rightarrow z_1(s)$; $\quad z_1(j\omega_0) = jX \qquad ; \; jX + j\omega_0 L = 0$

$z_1(s) + sL, \qquad L<0 \Rightarrow L = -X/\omega_0$

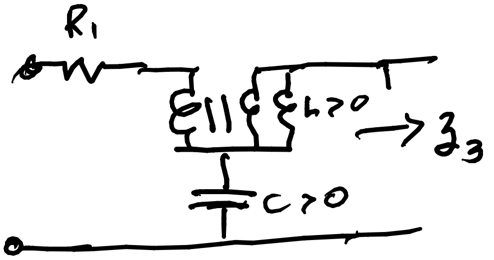
$z_1(s) = z(s) - sL$ is PR as $L<0$

$$z_1(j\omega_0) = 0 \Rightarrow \frac{1}{z_1(s)} = \frac{2k_1 s}{s^2+\omega_0^2} + \underbrace{y_2(s)}_{\text{PR \& a zero at } \infty}$$

$$\Rightarrow z_2(s) = \frac{1}{y_2(s)} = L_2 s + \underbrace{z_3(s)}_{\text{PR \& smallest}}$$



$\Downarrow$



Brune's uses coupled coils $\equiv$ using transformers gives synthesis of PR functions by passive components.