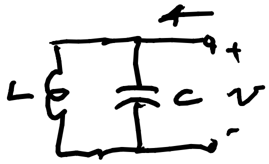
EE 610
11/08/12

Note Title 11/8/2012

p. 455 = limit cycle



$$y(s) = \frac{1}{Ls} + Cs = \frac{1+LCs^2}{Ls} = \frac{i}{v}$$

$$(1+LCs^2)v = Ls\,i$$

$= 0$ if open circuit

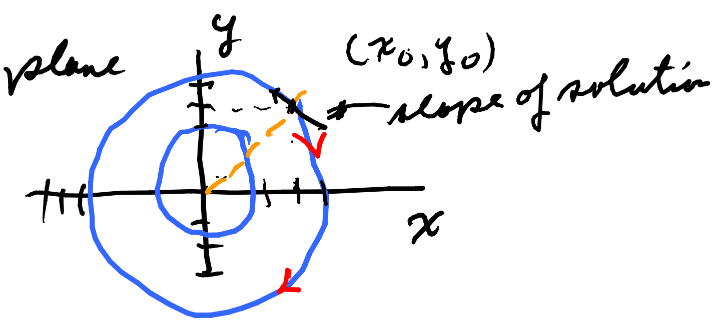
$$LC\frac{d^2v}{dt^2} + v = 0$$

$$\frac{LC}{t^2} \approx \frac{1}{\tau^2} \Rightarrow \tau = \frac{1}{\sqrt{LC}}\,t \Rightarrow \frac{d^2v}{d\tau^2} + v = 0$$

$x = v$, $y = \dot{x}$ $\qquad \dot{x} = y$

$\dot{y} = \ddot{x} = \ddot{v} = -v = -x$ $\qquad \dot{y} = -x$

phase plane



$$\frac{\dot{y}}{\dot{x}} = \frac{-x}{y} = \underbrace{\frac{dy}{dx}}$$

Van der Pol oscillator

$$\ddot{x} + \underbrace{\epsilon(x^2-1)\dot{x}}_{\frac{dF(x)}{dt}} + \omega_0^2 x = 0$$

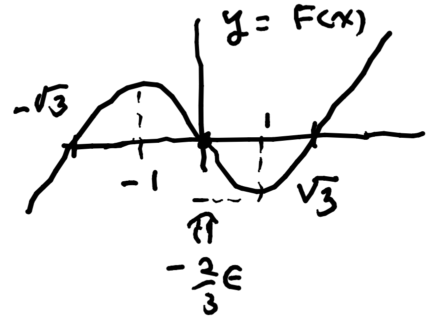
$$\Rightarrow F(x) = \epsilon\left(\frac{x^3}{3} - x\right) = \frac{\epsilon}{3}x(x^2-3)$$

$$\frac{d}{dt}(\dot{x} + F(x)) + x = 0 \qquad \text{by normalizing } \omega_0^2 = 1 \text{ via } t$$

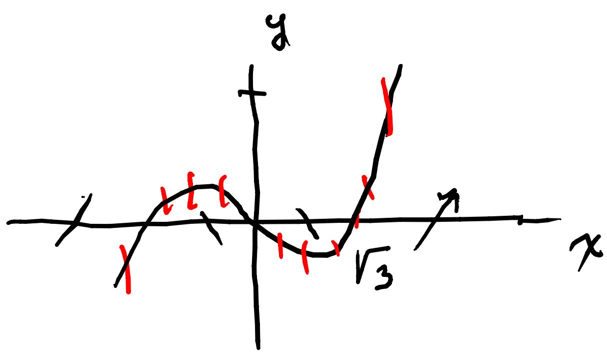
$$\frac{dy}{dt} = -x, \quad y = \dot{x} + F(x) \qquad \frac{dx}{dt} = y - F(x)$$

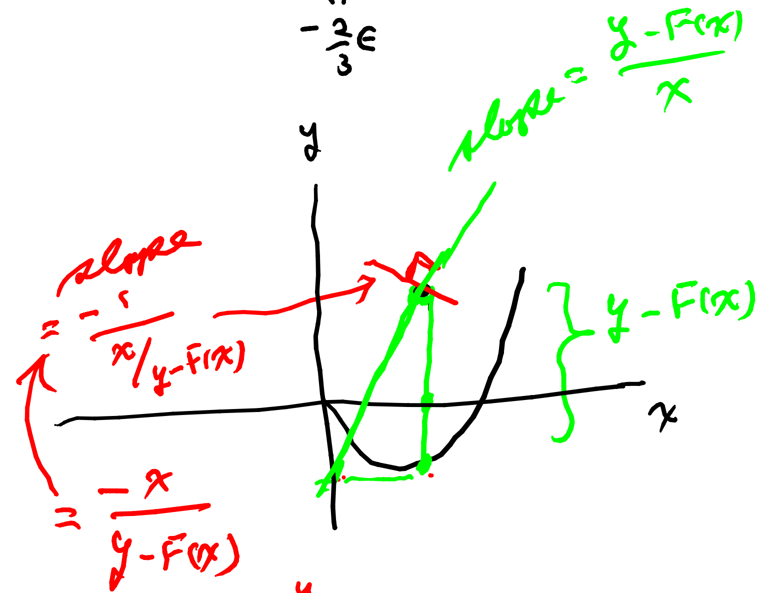
$$\frac{dy}{dt} = -x$$

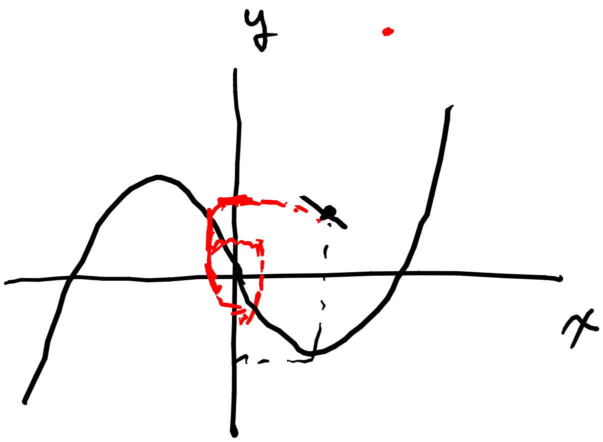
$$\frac{dy}{dx} = \frac{-x}{y - F(x)}$$

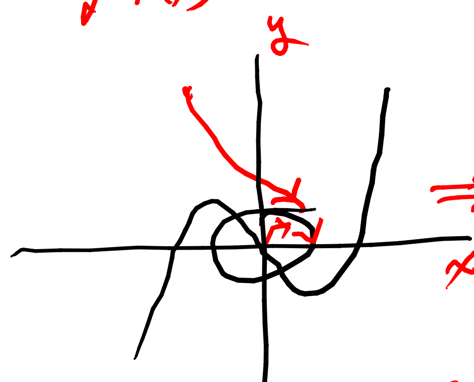


$$\frac{dF(x)}{dx} = \epsilon(x^2 - 1)$$
$$= 0 \;@\; x^2 = 1$$







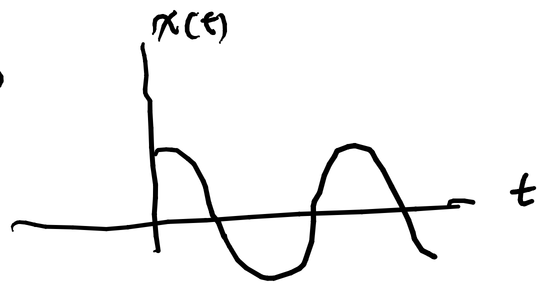


$\Rightarrow$ there is a limit cycle unique if $\epsilon > 0$

$\Downarrow$

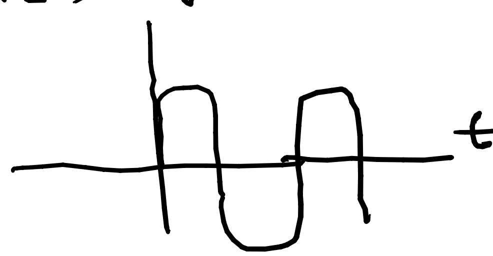
the Van der Pol oscillator is structurally stable

for $\epsilon \to 0$



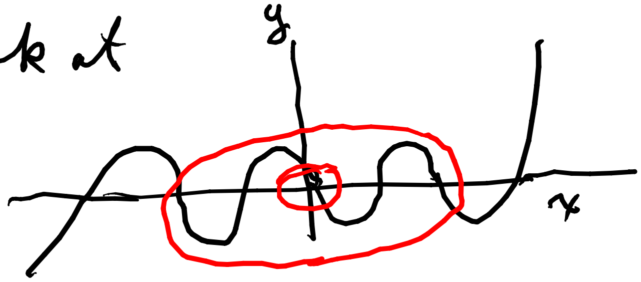
sine wave oscillator

for $\epsilon \to$ large > 0



relaxation oscillator

Look at



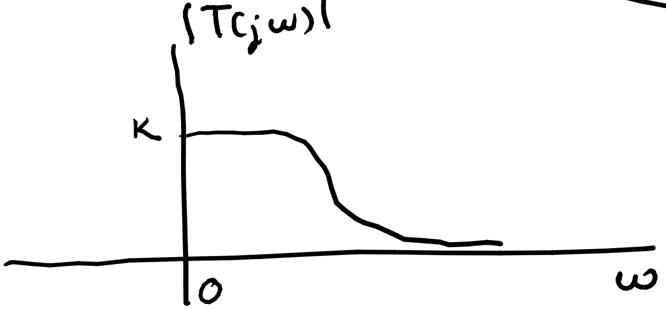
$\Rightarrow$ can make oscillators with more than one limit cycle

can get from resonant tunnel diodes

approximation for low pass filters

$$T(s) = \frac{v_o}{v_i}(s) = \frac{K}{s^n + a_{n-1} s^{n-1} + \cdots + a_2 s^2 + a_1 s + 1}$$

can normalize both to 1



desire "maximally flat"
$\Rightarrow$ as many derivatives of $|T(j\omega)|$ as possible are $= 0$

note $\dfrac{d|T(j\omega)|^2}{d\omega} = 2|T(j\omega)| \cdot \dfrac{d|T(j\omega)|}{d\omega}$

$$\frac{d\frac{1}{|T(j\omega)|}}{d\omega} = -\frac{1}{|T(j\omega)|^2} \cdot \frac{d|T(j\omega)|}{d\omega}$$

$\Rightarrow$ can set the derivatives of $\dfrac{1}{|T(j\omega)|^2} \Rightarrow 0$

$$\frac{1}{|T(j\omega)|^2} = \left(1 + a_1(j\omega) + a_2(j\omega)^2 + \cdots (j\omega)^n\right) \times \left(1 - a_1(j\omega) + a_2(-j\omega)^2 + \cdots (-j\omega)^n\right)$$

$$= 1 + c_1\omega^2 + \cdots + c_n\omega^{2n}$$ as this is even in ω

$\Rightarrow$ as this a Taylor series the coefficients are derivatives of $\dfrac{1}{|T(j\omega)|^2}$ & desire as many as possible to be zero

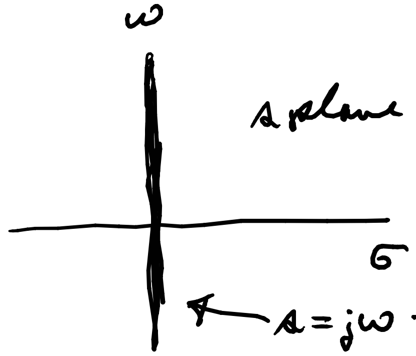
$$\frac{1}{|T(j\omega)|^2} = 1 + c_n\omega^{2n}$$

where $c_n\omega^{2n} = (j\omega)^n(-j\omega)^n$
$= j^{2n}(-1)^n \cdot \omega^{2n}$
$= (-1)^n(-1)^n\omega^{2n}$
$= \omega^{2n}$

$$= 1 + \omega^{2n}$$

$$= \frac{1}{T(j\omega)} \times \frac{1}{T(j\omega)^*}$$

$$= \frac{1}{T(j\omega)} \times \frac{1}{T(-j\omega)}$$ if real coefficients (rational if $n < \infty$)



$s = j\omega \Rightarrow \omega = s/j$ can use for all s

$$1 + \omega^{2n} \Big|_{\omega = s/j} = \frac{1}{T(j\omega)} \times \frac{1}{T(-j\omega)} \Big|_{\omega = s/j} = 1 + \left(\frac{s}{j}\right)^{2n} = 1 + (-1)^n s^{2n} = \frac{1}{T(s)} \times \frac{1}{T(-s)}$$

$\therefore$ desire factors of $P(s) = 1 + (-1)^n s^{2n} = D(s)D(-s)$

desire zeros of $(-1)^n s^{2n} + 1 \Rightarrow (-1)^n s^{2n} = -1$

$$\Rightarrow s^{2n} = (-1)^{n+1}$$

but $-1 = e^{j\pi} = e^{j(\pi + 2k\pi)}$ $k = 0, \pm 1, \pm 2, \ldots$

$$\left.\begin{array}{l} \text{if } n = \text{even}, (-1)^{n+1} = -1 \Rightarrow -1 = e^{j(\pi + 2k\pi)} \\ \text{if } n = \text{odd } (-1)^{n+1} = +1 \Rightarrow +1 = e^{j2k\pi} \end{array}\right\} k = 0, \pm 1, \ldots$$

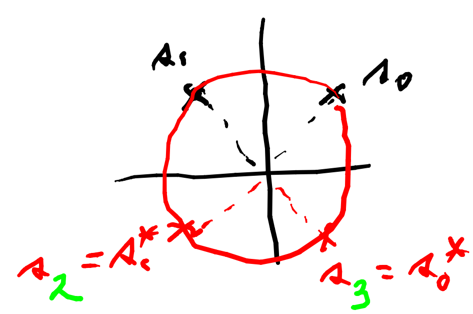
$\therefore$ the zeros of $\frac{1}{T(s)T(-s)}$ are $s_k = \begin{cases} e^{j(\pi + 2k\pi)/2n} & n \text{ even} \\ e^{j2k\pi/2n} & n \text{ odd} \end{cases}$ $k = 0, \pm 1, \ldots$

these are points on the unit circle

Ex: $n = 2$; s_0, s_1, s_2, s_3

$$s_0 = e^{j\pi/4}, \quad s_1 = e^{j(\pi + 2\pi)/4}$$

$$s_2 = e^{j(\pi + 4\pi)/4}$$



$$T(s)\cdot T(-s) = \frac{K^2}{(s-s_0)(s-s_1)(s-s_2)(s-s_3)}$$

for stable $T(s)$ choose left half plane poles, s_1, s_2

$$T(s) = \frac{K}{(s-s_1)(s-s_2)} = \frac{K}{\left(s-\left(-\frac{1}{\sqrt{2}}+j\frac{1}{\sqrt{2}}\right)\right)\left(s-\left(-\frac{1}{\sqrt{2}}-j\frac{1}{\sqrt{2}}\right)\right)}$$

$$= \frac{K}{s^2 + \frac{2}{\sqrt{2}}s + 1} \Rightarrow$$ 2nd order maximally flat transfer function