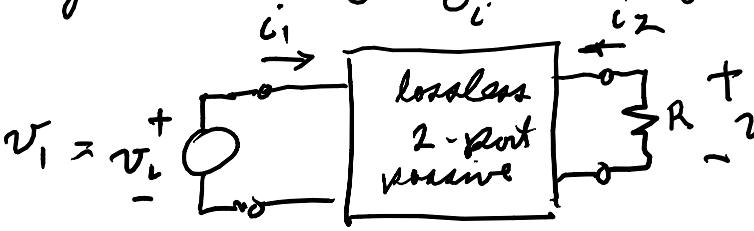
EE 610
10/25/12

Note Title 10/25/2012

Synthesis of $\frac{v_0}{v_i}(s)$ by terminated 2-port



$v_2 = v_0 = -R i_2 \Rightarrow i_2 = -G v_2$

$R = 1/G$

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$$i_2 = -G v_2 = y_{21} v_1 + y_{22} v_2 \Rightarrow -(G+y_{22}) v_2 = y_{21} v_1$$

$$\frac{v_2}{v_1} = \frac{-y_{21}}{G+y_{22}} = \frac{-y_{21}}{1+y_{22}}$$ if normalize $G=1$

∴ desire y_{22} = PR lossless = $\frac{n_{22}}{d_{22}}(s)$; n & d polynomial

as poles of y_{21} will need to be in poles of y_{22} then finite zeros all in y_{21}

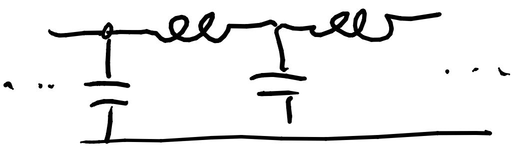
Ex: $$\frac{v_0}{v_i} = \frac{k}{s^3+3s^2+3s+1} = \frac{k}{(s^3+3s)+(3s^2+1)} =$$

$$= \frac{k/(3s^2+1)}{\left(\frac{s^3+3s}{3s^2+1}\right)+1} = \frac{k/(s^3+3s)}{1+\left(\frac{3s^2+1}{s^3+3s}\right)}$$ have 2 possible y_{22}

$y_{22_1} = \frac{s^3+3s}{3s^2+1}$, $y_{22_2} = \frac{3s^2+1}{s^3+3s}$

 $y_L = \frac{1}{sL} \Rightarrow$ open @ $s=\infty$

 $y_C = sC \Rightarrow$ short @ $s=\infty$



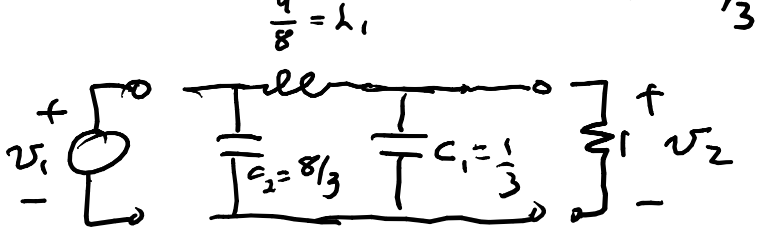
gives zeros of transmission at $s=\infty$

1st Cauer

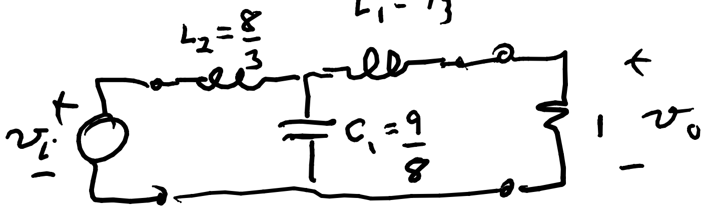
$\therefore$ desire a 1st Cauer synthesis of y_{22}

$$y_{22_1}=\frac{s^3+3s}{3s^2+1}$$

$$\begin{array}{l}
\qquad\qquad\quad \tfrac{1}{3}s \\
3s^2+1\,\overline{)\,s^3+3s} \\
\qquad\qquad s^3+\tfrac{1}{3}s \\
\qquad\qquad\overline{\qquad\quad}\ \ \tfrac{9}{8}s \\
\qquad\qquad \tfrac{8}{3}s\,\overline{)\,3s^2+1} \\
\qquad\qquad\qquad\ \ 3s^2 \qquad \tfrac{8}{3}s \\
\qquad\qquad\qquad\qquad\ 1\,\overline{)\,\tfrac{8}{3}s}
\end{array}$$

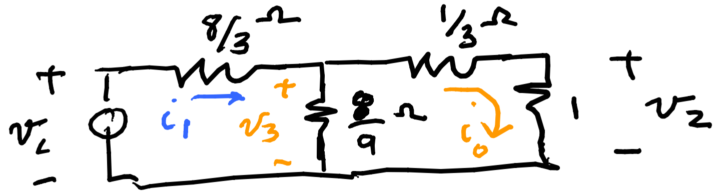


$$y_{22_2}=\frac{3s^2+1}{s^3+3s}=\cfrac{1}{\frac{1}{3}s+\cfrac{1}{\frac{9}{8}s+\cfrac{1}{\frac{8}{3}s}}}$$



$= \frac{v_0}{v_i}=\frac{1}{s^3+3s^2+3s+}$

$$\frac{v_0}{v_i}=\frac{k}{s^3+3s^2+3s+1}\quad @\ s=1 = \frac{v_0}{v_i}=\frac{k}{8}$$

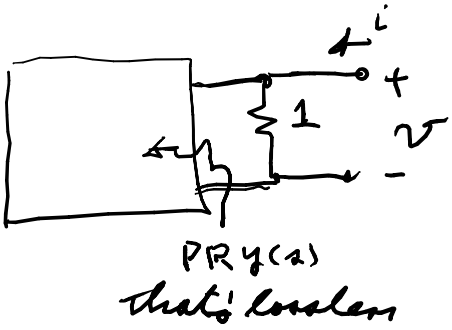


assume $v_2=1v$, $i_0=1a \Rightarrow v_3=1+\frac{1}{3}=\frac{4}{3}v$

$i_1=\frac{9}{8}\cdot\frac{4}{3}+1=\frac{3}{2}+1=5/2 \Rightarrow v_i=\frac{8}{3}\cdot i_1+v_3$

$v_i=\frac{8}{3}\cdot 5/2+\frac{4}{3}=\frac{20}{3}+\frac{4}{3}=24/3$ for $v_2=1$

$\therefore$ for $\frac{v_2}{v_1} = \frac{1}{24/3} = \frac{3}{24} = \frac{1}{8} = \frac{k}{8} \Rightarrow k = 1$



$i = (1+y)\cdot v \Rightarrow v = \frac{i}{1+y}$

$\Rightarrow$ natural response for $i=0$ is due to zeros of $1+y$

means all zeroes of $1+y$ are in the open left half plane

$1 + \frac{n}{d} = \frac{d+n}{d} \Rightarrow$ zeros of $1+y$ are zeroes of $n+d$ of y

$\therefore$ if y is PR lossless then for $y = \frac{n}{d}$ we have $P(s) = n(s) + d(s)$ is a Hurwitz polnomial $\Leftrightarrow$ no zeroes in $\sigma \geq 0$

$\Rightarrow$ gives a nice test for Hurwitz polynomials $P(s) = \text{Ev}\,P(s) + \text{Od}\,P(s)$

$y = 1 + \frac{\text{Od}\,P}{\text{Ev}\,P}$ or $1 + \frac{\text{Ev}\,P}{\text{Od}\,P}$

$P(s) = s^5 + 4s^4 + 3s^3 + 2s^2 + s + 1$

$(s^5 + 3s^3 + s) + (4s^4 + 2s^2 + 1)$

$$\frac{s^5 + 3s^3 + s}{4s^4 + 2s^2 + 1} = y(s)$$

$\frac{1}{4}s$

$4s^4 + 2s^2 + 1 \overline{\big)\, s^5 + 3s^3 + s}$

$s^5 + \frac{1}{2}s^3 + \frac{1}{4}s$

$\frac{8}{5}s$

$\frac{5}{2}s^3 + \frac{3}{4}s \overline{\big)\, 4s^4 + 2s^2 + 1}$

$4s^4 + \frac{6}{5}s^2$

$\frac{25}{8}s$

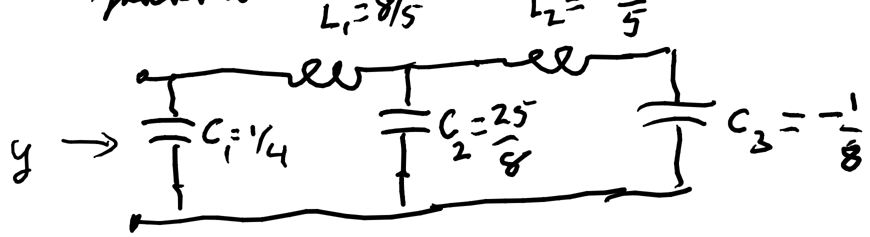
$\frac{4}{5}s^2 + 1 \overline{\big)\, \frac{5}{2}s^3 + \frac{3}{4}s}$

$\frac{5}{2}s^3 + \frac{25}{8}s$

$-\frac{1}{8}s \overline{\big)\, 4/5\, s^2 + 1}$

The Cauer synthesis is not passive

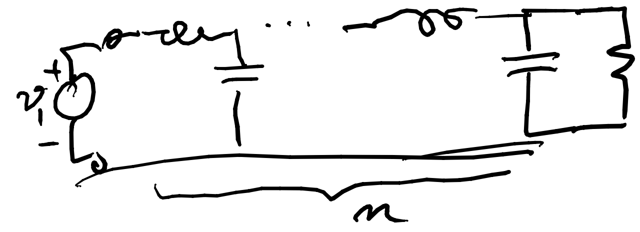
$$= -\frac{32}{5}s + \frac{1}{-\frac{1}{8}s}$$



--- check that can use one of the two y_{22}'s ⇒ od/Ev or Ev/od

$\frac{v_2}{v_1}$ all zeroes of transmission @ ∞ = $\frac{k}{s^n + \cdots}$

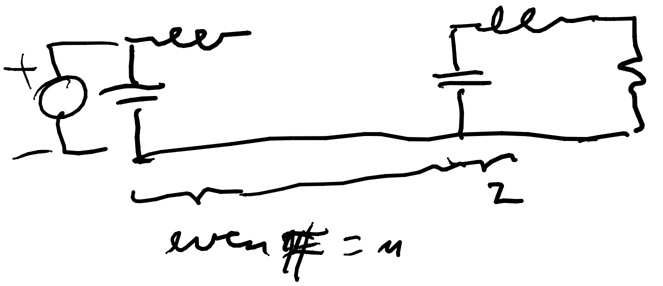
n is even



$y_{22} = \frac{Ev\,P}{Od\,P}$ ⇒ pole@∞ works

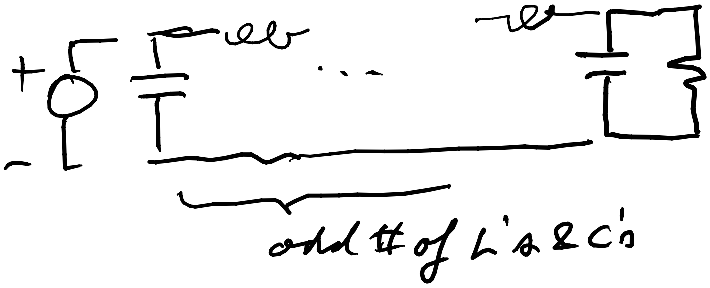
$y_{22} = \frac{Od\,P}{even}$ ⇒ 0@∞

doesn't work



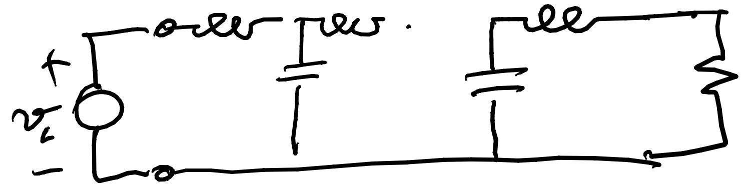
n is odd

$y_{22} = \frac{Od\,P}{Ev\,P}$ pole@∞



doesn't work

$y_{22} = \frac{Ev\,P}{Od\,P}$ 0 at ∞



works

$\delta\left(\frac{v_2}{v_1}\right) = n$

these work for low-pass $v_o/v_i(s)$, terminated in R