EE 610

10/23/12

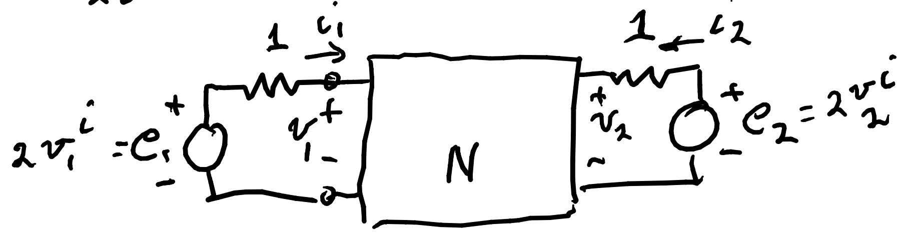
Scattering matrix, p. 177

$2v^i = v + z_0 i$ normalize z_0 to 1_2 for a 2-port

$2v^r = v - z_0 i$

$2v^i = v + i$

$2v^r = v - i$



$e = v + i = 2v^i$

$2v^r = 2v$ if $e = 0$

$E_1 = 2V_1^i$; $E_2 = 0 \Rightarrow V_2 = V_2^r = \dfrac{V_2}{E_1} = \dfrac{V_2^r}{2V_1^i}\Bigg|_{V_2^i = 0} = \dfrac{S_{21}}{2}$ S_{21} = transmission coefficient

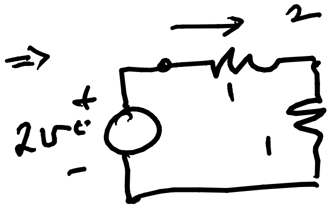
$V^r = S\,V^i$

gives physical meaning of S_{21} as a voltage transfer ratio

$S_{11} = \dfrac{V_1^r}{V_1^i}\Bigg|_{V_2^i = 0}$ = reflection coefficient at port 1 when terminate at port 2 in $z_0 = 1$

$i = \frac{1}{2} \cdot 2v^i = v^i$

If $y_{11} = 1$, other y's $= 0 \Rightarrow$



$S_{11} = \dfrac{v_1^r}{v_1^i} = \dfrac{v - v}{v^i} = 0$

$V^r = S(s)V^i \Rightarrow s = j\omega \Rightarrow$ Laplace transforms = Fourier transforms

passive case $\mathcal{E}(t) = \int_{-\infty}^{t} p(\tau)\,d\tau \geq 0 \Rightarrow$ also if $t \to \infty$

$\mathcal{E}(\infty) = \int_{-\infty}^{\infty} v^T(\tau)\,i(\tau) \geq 0$

$v = v^i + v^r$, $i = v^i - v^r$

$$v^T i = [v^i + v^r]^T [v^i - v^r] = v^{iT} \cdot v^i - v^{rT} \cdot v^r + \underbrace{v^{rT} \cdot v^i - v^{iT} v^r}_{\text{sum} = 0}$$

$$\mathcal{E}(\infty) = \int_{-\infty}^{\infty} [v^{iT}(t) \cdot v^i(t) - v^{rT}(t)\, v^r(t)]\, dt \geq 0 \text{ for passive}$$

$\int_{-\infty}^{\infty} v_1^i(t)\, v_1^i(t)\, dt$ is one component to look at

Plancherel theorem

$$\int_{-\infty}^{\infty} f(t) g(t)\, dt = \int_{-\infty}^{\infty} \mathcal{F}^*[f(t)]\, \mathcal{F}[g(t)]\, df$$

$$\mathcal{F}[f] = \int_{-\infty}^{\infty} f(t) e^{-j2\pi f t}\, df, \quad g(t) = \mathcal{F}^{-1}[\mathcal{F}[g]] = \int_{-\infty}^{\infty} \mathcal{F}[g] e^{+j2\pi f t}\, df$$

$$\Rightarrow \int_{-\infty}^{\infty} v_1^i(t)\, v_1^i(t)\, dt = \int_{-\infty}^{\infty} V_1^{i*}(j\omega)\, V_1^i(j\omega)\, df \quad f = \omega/2\pi$$

$$\int_{-\infty}^{\infty} p(t)\, dt = \int_{-\infty}^{\infty} [v^{iT}(t) \cdot v^i(t) - v^{rT}(t)\, v^r(t)]\, dt$$

$$= \int_{-\infty}^{\infty} [V^{iT*} V^i - V^{rT*} V^r] \frac{d\omega}{2\pi} > 0 \Rightarrow \int_{-\infty}^{\infty} v^{iT} v^i > \int v^{rT} v^r$$

$$\Rightarrow \int_{-\infty}^{\infty} [V^{iT*} V^i - V^{iT*} S^{T*}(j\omega)\, S(j\omega)\, V^i]\, df \geq 0$$

$$= \int_{-\infty}^{\infty} V^{iT*} [1_2 - S^{T*}(j\omega)\, S(j\omega)]\, V^i\, df \geq 0$$

implies that $1_2 - S^{T*}(j\omega)\, S(j\omega)$ is a positive semidefinite

$\Rightarrow S(s)$ has no poles on $s = j\omega$

If $\mathcal{E}(\infty) = 0 \Rightarrow$ a lossless circuit $\Rightarrow 1_2 = S^{T*}(j\omega)\, S(j\omega)$

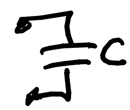
$\Rightarrow$ lossless $S^{-1}(j\omega) = S^{T*}(j\omega)$

If $S(j\omega)$ is rational in $j\omega$ with real coefficients (a real circuit) then $S^{*}(j\omega) = S(-j\omega)$

now replace $j\omega$ by $s = j\omega$, $\Rightarrow \omega = s/j$

$\Rightarrow$ lossless $\Rightarrow$ $1_2 = S^{T}(-s)S(s)$ for all s; this analytically continues S from $j\omega$ axis to whole s-plane

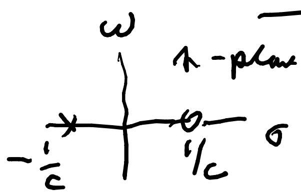
Ex: $S(s)$ for a capacitor; $y(s) = sC$



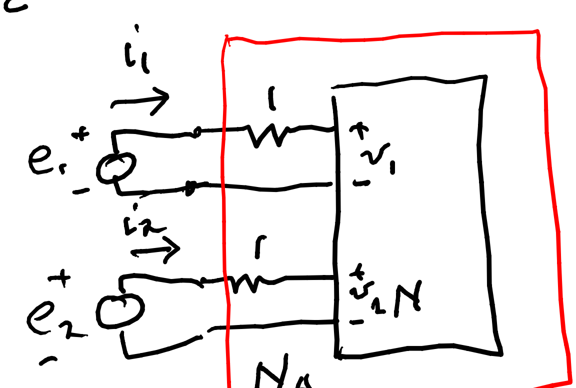
We have $v^r = S(s)v^i$

$$\left[\frac{v-i}{2}\right] = S(s)\left[\frac{v+i}{2}\right]$$

$$\frac{v - y\cdot v}{2} = S(s)\left[\frac{v+yv}{2}\right] \Rightarrow S(s) = \frac{1-y}{1+y} = \frac{1-sC}{1+sC}$$



$$S(-s) = \frac{1-(-s)C}{1+(-s)C} = \frac{1+sC}{1-sC} = 1/S(s)$$



N_A = augmented N

$i = Y_A \cdot e$; $i = v_0 - v_1$, $e = 2v^i$

$$(v^i - v^r) = Y_a \cdot 2v^i \Rightarrow (1_2 - 2Y_a)v^i = v^r$$

$S = 1_2 - 2Y_a$ $\Rightarrow$ gives a nice way to find S

Here $\int v_1^i(t)\, v_1^i(t)\, dt \geq 0$ if finite $\Rightarrow$ call v_1^i square integrable, it is in $\mathcal{L}_b^2 = \mathcal{L}\text{-}2$, Lebesgue integrable

if we apply $v^i \in \mathcal{L}_b^2$

then $v^n \in \mathcal{L}_b^2$

as $v = v^i + v^n \Rightarrow v \in \mathcal{L}_b^2$, & $i \in \mathcal{L}_b^2$ if N is passive

$i = v^i - v^n$

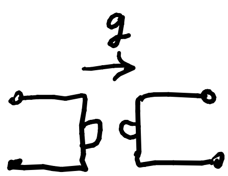
finite energy in v^i gives finite energy in v^n & v, i

Here for N passive, $S(s)$ is bounded real

if rational we call it BR: no poles in $\sigma \geq 0$

& $1_2 - S^T(-s)S(s)\big|_{s=j\omega} \geq 0$

& only real coefficients

Ex: 

$; \quad Y = \begin{bmatrix} 0 & -g \\ g & 0 \end{bmatrix}, \quad S = (1_2 - Y)(1_2 + Y)^{-1}$

$$S = \begin{bmatrix} 1 & +g \\ -g & 1 \end{bmatrix} \begin{bmatrix} 1 & -g \\ g & 1 \end{bmatrix}^{-1}, \quad \begin{bmatrix} 1 & -g \\ g & 1 \end{bmatrix}^{-1} = \frac{1}{1+g^2} \begin{bmatrix} 1 & (-1)^{2+1}(-g) \\ (-1)^{1+2}(g) & 1 \end{bmatrix}$$

$$= \frac{1}{1+g^2} \begin{bmatrix} 1 & g \\ -g & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & +g \\ -g & 1 \end{bmatrix}^2 \times \frac{1}{1+g^2}$$

$$= \frac{1}{1+g^2} \begin{bmatrix} 1-g^2 & 2g \\ -2g & 1-g^2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & g \\ -g & 1 \end{bmatrix} \begin{bmatrix} 1 & g \\ -g & 1 \end{bmatrix} = \begin{bmatrix} 1-g^2 & 2g \\ -2g & 1-g^2 \end{bmatrix}$$

Note $S = (1_2 - Y)(1_2 + Y)^{-1} \underset{\uparrow \text{true}}{\overset{?}{=}} (1_2 + Y)^{-1}(1_2 - Y)$

$$(1_2 + Y)(1_2 - Y) = 1_2 - Y^2 = (1_2 - Y)(1_2 + Y)$$

If $Y = Y^T$ then $S = S^T$ since $S^T = (1_2 + Y^T)^{-1}(1 - Y^T)$

$= (1 + Y)^{-1}(1 - Y) = S$