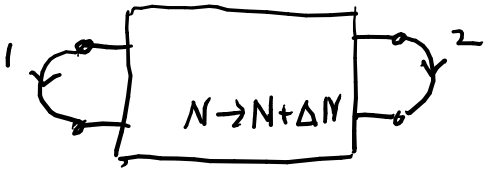
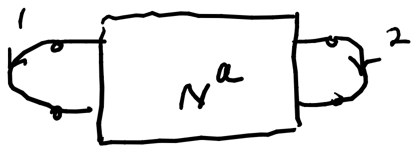
Note Title 10/10/2012

EE 610
10/18/12 → 11/06/12

$$\frac{\partial T(s)}{\partial x} \Big/ \frac{T(s)}{x} = S_x^{T(s)} = \text{sensitivity of } T \text{ with respect to } x$$





N^a = adjoint
same graph as N

$$v_b = \begin{bmatrix} v_1 \\ v_2 \\ \hline v \end{bmatrix} \qquad i_b = \begin{bmatrix} i_1 \\ i_2 \\ i \end{bmatrix}$$

assume $Y_N = Y$ exists
as does $Y_{N^a} = Y^a$

Have i_b, v_b, i_b^a, v_b^a $\qquad i_b^T v_b^a - i_b^{aT} v_b = 0$

$$i_1 \cdot v_1^a - i_1^a v_1 + i_2 \cdot v_2^a - i_2^a v_2 + v^T Y^T v^a - v^{aT} Y^{aT} v$$

Make a change in N, Δ; but dont change in N^a

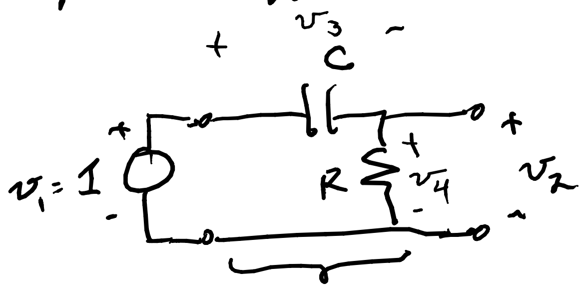
$$\Delta i_1 \cdot v_1^a - i_1^a \Delta v_1 + \Delta i_2 \cdot v_2^a - i_2^a \cdot \Delta v_2 + (\Delta v^T Y^T v^a - v^{aT} Y^{aT} \Delta v) + v^T \Delta Y^T v^a = 0$$

If interested in $\frac{v_2}{v_1} = T(s)$ then fix v_1 & see how v_2 changes, look at Δv_2 (as change something in N)
$\Rightarrow \Delta v_1 = 0$; as want Δv_2, isolate it by $i_2^a = 1$
set $v_1^a = 0 \Rightarrow \Delta i_1 v_1^a = 0$; force $\Delta i_2 = 0$ by open circuit on port 2 of N

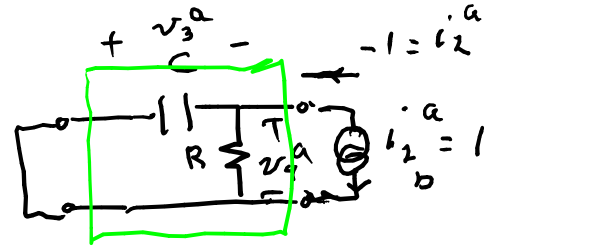
force
$$(\Delta v^T Y^T v^a - v^{aT} Y^{aT} \Delta v) = 0 = \Delta v^T Y^T v^a - \Delta v^T Y^a v^a$$
$$= \Delta v^T (Y^T - Y^a) v^a \Rightarrow Y^T - Y^a = 0_{(b-2)\times(b-2)}$$
↑
since Y's exist, holds for all v^a & v

⇒ desire $Y^a = Y^T$ = transpose Y of original

∴ just analyze the circuits to get $\Delta v_2/\Delta x\big|_{v_1=1} = \frac{\Delta T(x)}{\Delta x}$, x = component



$\frac{\partial\, v_2/v_1}{\partial C}$



$Y = \begin{bmatrix} sC & -sC \\ -sC & sC+G \end{bmatrix}$, $G = 1/R$

$Y^a = Y^T = \begin{bmatrix} sC & -sC \\ -sC & sC+G \end{bmatrix}$

if change c; $\Delta Y = \begin{bmatrix} s\Delta C & -s\Delta C \\ -s\Delta C & s\Delta C \end{bmatrix} = s\Delta C \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

know $-\Delta v_2 = -v^T \Delta Y v_a = -s\Delta C \, [v_3 \;\; v_4] \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} v_3^a \\ v_4^a \end{bmatrix}$

for N; $v_1 = 1$: $v_4 = \frac{R}{R + \frac{1}{sC}} \cdot v_1$, $v_3 = \frac{1/sC}{R + \frac{1}{sC}} \cdot v_1$; $v_3^a = -v_4^a$

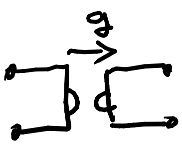
$v_4^a = \frac{1}{G + sC} \cdot (-1)$

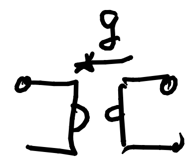
$-\Delta v_2 = -s\Delta C \left[\frac{RsC}{1+RsC}, \; \frac{1}{1+RsC} \right] \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{G+sC} \\ -\frac{1}{G+sC} \end{bmatrix}$

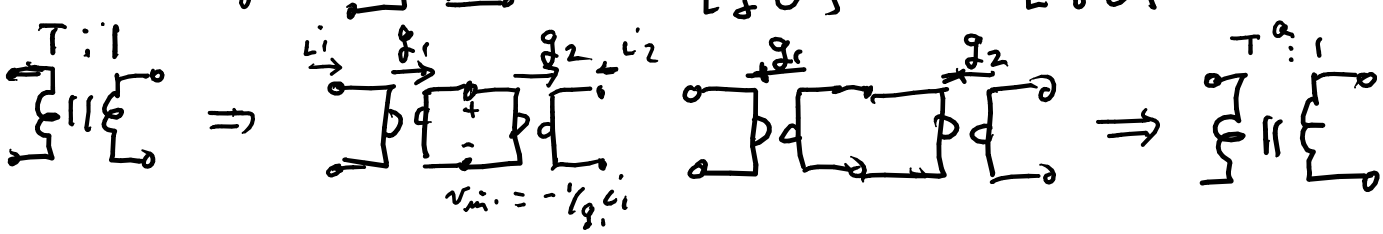
$= -s\Delta C \, \frac{1}{1+RsC} \cdot \frac{1}{G+sC} \left[RsC - 1, \; -RsC + 1 \right] \begin{bmatrix} 1 \\ -1 \end{bmatrix} =$

$= -\frac{2(RsC-1)}{(1+RsC)(G+sC)} \quad (s\Delta C)$

⇒ $\frac{\Delta v_2}{\Delta c} = \frac{\partial v_2}{\partial C} = \frac{2s(RsC-1)}{(RsC+1)^2} \cdot R = \frac{\partial (v_2/v_1)}{\partial C}$

adjoint of 

⇒ $Y = \begin{bmatrix} 0 & -g \\ g & 0 \end{bmatrix}$, $Y^a = \begin{bmatrix} 0 & g \\ -g & 0 \end{bmatrix}$ ⇒ 



$v_1 = v_{mid} = -\frac{1}{g_1} i_1$ $\qquad T^a = T = g_2/g_1$

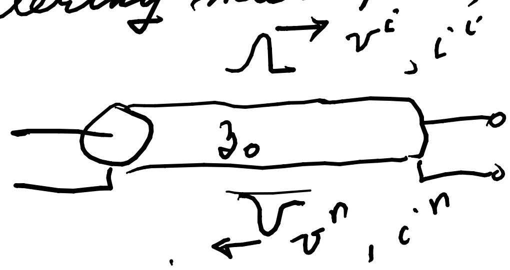
$i_2 = g_2 \cdot v_{mid}$

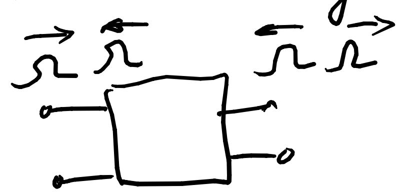
$\frac{-i_1}{g_1} = \frac{i_2}{g_2} = -\frac{g_2}{g_1} i_1 = i_2$

$v_2 = \frac{g_2}{g_1} v_1 \Rightarrow T = \frac{g_2}{g_1}$

$Y = \begin{bmatrix} 0 & 0 \\ g_m & 0 \end{bmatrix}$ $\qquad Y^a = \begin{bmatrix} 0 & g_m \\ 0 & 0 \end{bmatrix} = Y^T$

scattering matrix ⇒ waves; incident & reflected





$\cancel{2} v \triangleq v^i + v^r$ $\quad\Rightarrow\quad 2v^i = \cancel{2} v + \cancel{2} Z_0 i \Rightarrow 2v^i = v + Z_0 i$

$\cancel{2} Z_0 i \triangleq v^i - v^r$ $\qquad\qquad 2v^r = \cancel{2} v - \cancel{2} Z_0 i \qquad 2v^r = v - Z_0 i$

normally $Z_0 = 50\,\Omega$ ⇒ normalize to 1 ⇒ $2v^i = v + i$, $2v^r = v - i$

like direct & quadrature axis voltages of power systems.

Define scattering matrix by $v^r = S v^i$

but $v^i = (1_2 + Y)v$, $v_r = (1_2 - Y)v$

$\Rightarrow v = (1_2 + Y)^{-1} v_i \Rightarrow v_r = (1_2 - Y)(1_2 + Y)^{-1} v^i$

$$S = (1_2 - Y)(1_2 + Y)^{-1}$$

$y = sC$ $\qquad S = (1-y)/(1+y) = \frac{1 - sC}{1 + sC}$; here $S^{-1}(s) = S(-s)$

general property of lossless circuits

Look at $v^T i = \frac{1}{2}(v^i + v^r)^T \left(\frac{1}{2}[v^i - v^r]\right)$