EE 610
10/16/12

Note Title 10/10/2012

$$Y = \begin{bmatrix} 0 & -g \\ g & 0 \end{bmatrix} ; \quad \det Y = \begin{cases} g^2 \\ 0 \text{ if } g = 0 \end{cases} \quad \text{rank} = 2$$

$$Y = \begin{bmatrix} 0 & -g_1 & -g_2 \\ g_1 & 0 & -g_3 \\ g_2 & g_3 & 0 \end{bmatrix} ; \quad \det Y = -g_1 \begin{vmatrix} -g_1 & -g_2 \\ g_3 & 0 \end{vmatrix} + g_2 \begin{vmatrix} -g_1 & -g_2 \\ 0 & -g_3 \end{vmatrix}$$

$$= -g_1 g_2 g_3 + g_2(g_1 g_3) = 0$$

but $\begin{vmatrix} 0 & -g_1 \\ g_1 & 0 \end{vmatrix} = g_1^2 \Rightarrow$ if any $g_i \neq 0$ rank $Y = 2$

$$Y = \begin{bmatrix} 0 & -g_1 & 0 \\ g_1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & -g_2 \\ 0 & 0 & 0 \\ g_2 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -g_3 \\ 0 & g_3 & 0 \end{bmatrix}$$

Y

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -\frac{g_2}{g_1} & 1 \end{bmatrix} \begin{bmatrix} 0 & -g_1 & -g_2 \\ g_1 & 0 & -g_3 \\ g_2 & g_3 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -\frac{g_2}{g_1} \\ 0 & 0 & 1 \end{bmatrix} \qquad T_1 Y T_1^T$$

$$\| $$

$$\begin{bmatrix} 0 & -g_1 & -g_2 \\ g_1 & 0 & -g_3 \\ 0 & g_3 & +\frac{g_3 g_2}{g_1} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -\frac{g_2}{g_1} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -g_1 & 0 \\ g_1 & 0 & -g_3 \\ 0 & g_3 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ +\frac{g_3}{g_1} & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -g_1 & 0 \\ g_1 & 0 & -g_3 \\ 0 & g_3 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & g_3/g_1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad T_2 (T_1 Y T_1^T) T_2^T = Y_{g_1}$$

$$= \begin{bmatrix} 0 & -g_1 & 0 \\ g_1 & 0 & -g_3 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & g_3/g_1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -g_1 & 0 \\ g_1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$T_1 Y T_1^T = (T_2^{-1} Y_{g_1} T_2^{-T})$$

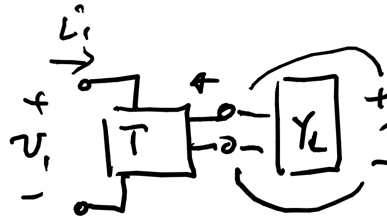
$$Y = T_1^{-1} \underbrace{(T_2^{-1} Y_{g_1} T_2^{-T})}\, T_1^{-T}$$

$$Y = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & \frac{g_2}{g_1} & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -\frac{g_3}{g_1} & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -g_1 & 0 \\ g_1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & -g_3/g_1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & g_2/g_1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -\frac{g_3}{g_1} & \frac{g_2}{g_1} & 1 \end{bmatrix} \underbrace{\begin{bmatrix} 0 & -g_1 & 0 \\ g_1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}}_{Y_L} \begin{bmatrix} 1 & 0 & -g_3/g_1 \\ 0 & 1 & g_2/g_1 \\ 0 & 0 & 1 \end{bmatrix} = T Y_{g_1} T^T$$

$$\Downarrow$$

$$T = T_1^{-1} T_2^{-1}$$



$$v_2 = T^T v_1$$

$$i_1^T v_1 + i_2^T v_2 = 0$$

$$i_1^T v_1 + i_2^T T^T v_1 = 0$$

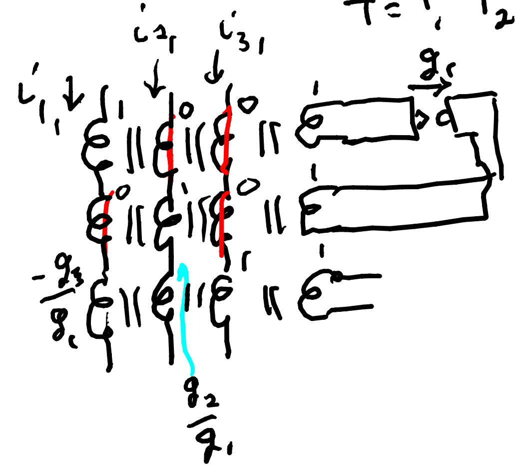
$$i_1^T = -i_2^T T^T$$

$$\Rightarrow i_1 = -T i_2$$

$$-i_2 = Y_L v_2$$

$$= Y_L T^T v_1$$

$$i_1 = T Y_L T^T$$



$$-QA - A^TQ = -\begin{bmatrix} q_{11} & c_0 \\ c_0 & c_1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ -a_0 & -a_1 \end{bmatrix} - \begin{bmatrix} 0 & -a_0 \\ 1 & -a_1 \end{bmatrix} \begin{bmatrix} q_{11} & c_0 \\ c_0 & c_1 \end{bmatrix}$$

$$= \begin{bmatrix} 2c_0a_0 & -q_{11} + c_1a_0 + a_1c_0 \\ -q_{11} + c_1a_0 + a_1c_0 & 2(a_1c_1 - c_0) \end{bmatrix}$$

choose $q_{11} = c_1a_0 + a_1c_0 \Rightarrow -QA - A^TQ = \underbrace{\begin{bmatrix} 2c_0a_0 & 0 \\ 0 & 2(a_1c_1 - c_0) \end{bmatrix}}$

$r_1 = 1/2 c_0 a_0$

$r_2 = \dfrac{1}{2(a_1c_1 - c_0)}$

$$y(s) = \frac{c_0 + c_1 s}{s^2 + a_1 s + a_0}$$

$$\begin{array}{r} \frac{1}{c_1} s \\ c_1 s + c_0 \overline{\big)\, s^2 + a_1 s + a_0} \\ s^2 + \frac{c_0}{c_1} s \\ \hline \frac{a_1c_1 - c_0}{c_1} s \end{array}$$

by PR ≥ 0

$$Q = \begin{bmatrix} c_1a_0 + c_0a_1 & c_0 \\ c_0 & c_1 \end{bmatrix}$$

Next factor $Q = T^T T$; force (1,2) term to zero by $\begin{bmatrix} 1 & -c_0/c_1 \\ 0 & 1 \end{bmatrix}$ on left & its transpose on right

$$\begin{bmatrix} 1 & -c_0/c_1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} c_1 a_0 + c_0 a_1 & c_0 \\ c_0 & c_1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -\frac{c_0}{c_1} & 1 \end{bmatrix} = \begin{bmatrix} c_1 a_0 + c_0 a_1 - c_0^2/c_1 & 0 \\ 0 & c_1 \end{bmatrix}$$

$$= \begin{bmatrix} c_1 a_0 + \frac{c_0}{c_1}(a_1 c_1 - c_0) & 0 \\ 0 & c_1 \end{bmatrix}$$

$$= \begin{bmatrix} \sqrt{c_1 a_0 + \frac{c_0}{c_1}(a_1 c_1 - c_0)} & 0 \\ 0 & \sqrt{c_1} \end{bmatrix} \begin{bmatrix} \sqrt{c_1 a_0 + \frac{c_0}{c_1}(a_1 c_1 - c_0)} & 0 \\ 0 & \sqrt{c_1} \end{bmatrix}$$

$$\Rightarrow Q = T^T T = \begin{bmatrix} c_1 a_0 + c_0 a_1 & c_0 \\ c_0 & c_1 \end{bmatrix} \Longrightarrow T = \underbrace{\begin{bmatrix} \sqrt{c_1 a_0 + \frac{c_0}{c_1}(a_1 c_1 - c_0)} & 0 \\ 0 & \sqrt{c_1} \end{bmatrix}}_{\text{entries} \geq 0 \text{ by PR condition}} \begin{bmatrix} 1 & 0 \\ \frac{c_0}{c_1} & 1 \end{bmatrix}$$

Then form $Y_1 = \begin{bmatrix} -TAT^{-1} & -TB \\ +CT^{-1} & 0 \end{bmatrix}$, realize $Y_{1,skew}$ by gyrators

form $T^T Y_{1,sym} T = \begin{bmatrix} g_1 & 0 & 0 \\ 0 & g_2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ &

realize $Y_{1\,sym}$ by transformer loaded by passive resistors, $r_1 = \frac{1}{2c_0 a_0}$, $r_2 = \frac{1}{2(a_1 c_1 - c_0)}$

connect $Y_{1\,skew}$ & $Y_{1\,sym}$ circuits in parallel and load first 2 ports in unit capacitors; 3rd port gives $y(s) = \frac{i_3}{v_3} = \frac{c_0 + c_1 s}{s^2 + a_1 s + a_0}$

$\Rightarrow$ can synthesize by a passive circuit any PR function