Passive: $p(t) = \text{power} = v(t)^T i(t) = [v_1 \;\; v_2]\begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$

sum power into ports

Energy $= \mathcal{E}(t) = \int_{-\infty}^{t} p(\tau)\, d\tau$

if $\mathcal{E}(t) \geq 0$ for all t then the circuit is passive

If $i(t) = Y \cdot v(t)$ then; $p(t) = v^T \cdot i = v^T Y v$

if $p(t) \geq 0$ then Y (a constant one) gives a passive circuit.

$p(t) = p^T(t)$ as a scalar

$= v^T Y v = v^T Y^T v$

$2p(t) = v^T(Y + Y^T)v \Rightarrow p(t) = v^T(Y_{sym})v$

$0 = p - p^T = v^T(Y - Y^T)v = 0 = v^T(Y_{skew})v$

$Y = \dfrac{Y + Y^T}{2} + \dfrac{Y - Y^T}{2} \Rightarrow$ no power used in skew symmetric part

if $p(t) > 0$ for all $v \neq 0$ then

$Y_{sym} > 0$ is called positive definite

$\geq$ " " positive semi-definite.

from mathematics know there is a real constant matrix T such that

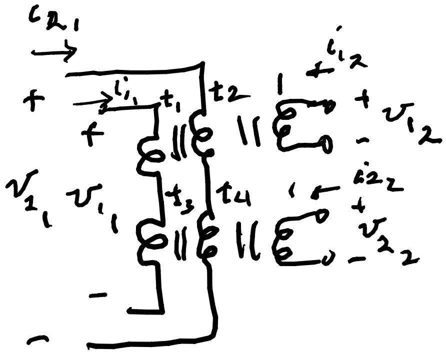
$T Y_{sym} T^T =$ diagonal matrix $= \begin{bmatrix} d_1 & & & \\ & d_2 & & \\ & & \ddots & \\ & & & d_n \end{bmatrix}$

and $d_i > 0$ if positive definite

$d_i \geq 0$ if " semi-definite

$Y_{sym} = (T^{-1}\sqrt{D})(\sqrt{D}\, T^{-T})$

$p(t) = v^T(T^{-1}\sqrt{D})(\sqrt{D}\, T^{-T} v) = x^T x$



$$v_{1_1} = t_1 v_{1_2} + t_3 v_{2_2}$$

$$v_{2_1} = t_2 v_{1_2} + t_4 v_{2_2}$$

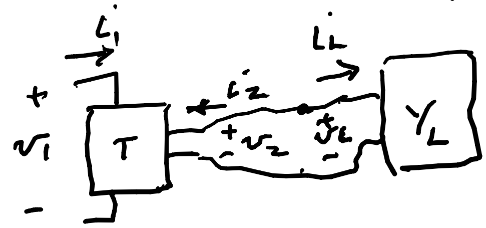
$$v_1 = \begin{bmatrix} t_1 & t_3 \\ t_2 & t_4 \end{bmatrix} v_2 = T^T v_2$$

$$P_{in} = 0 = v_1^T i_1 + v_2^T i_2 = v_2^T T i_1 + v_2^T i_2$$

$$= v_2^T (T i_1 + i_2) = 0 \text{ for all } v_2$$

$$i_2 = -T i_1$$

$$v_1 = T^T v_2$$



$$i_L = Y_L v_L$$

$$= -i_2, \quad v_L = v_2$$

$$-i_2 = Y_L v_2 = -(-T i_1) = Y_L (T^{-T}) v_1$$

$$i_1 = T^{-1} Y_L T^{-T} v_1$$

$$i_1 = Y v_1 = T^{-1} Y_L T^{-T} v_1$$

$$Y = T^{-1} Y_L T^{-T}$$

$$T Y T^T = Y_L$$

$$y(s) = \frac{\sum_0^{n-1} c_i s^i}{s^n + \sum_0^{n-1} a_i s^i} + g_0 + g_1 s + g_2 s^2 + \dots$$

here we note for g_0 $\Rightarrow$ $i = g_0 v$

$$0 \cdot \dot{x} = -x + v$$

$$i = g_0 x$$

$E\dot{x} = Ax + Bu$
$y = Cx$

$$\Rightarrow \begin{bmatrix} \dot{x}_1 \\ 0 \cdot \dot{x}_2 \end{bmatrix} = \begin{bmatrix} A & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} B \\ 1 \end{bmatrix} u; \quad y = \begin{bmatrix} C & g_0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

do the same for $g_1 s$, $g_2 s^2$, etc.

for $g_1 s$

$$\begin{bmatrix} 0 & 0 \\ s & 0 \end{bmatrix}\begin{bmatrix} x_{1_2} \\ x_{2_2} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x_{1_2} \\ x_{2_2} \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} c_{1_2} & c_{2_2} \end{bmatrix}\begin{bmatrix} x_{1_2} \\ x_{2_2} \end{bmatrix}$$

desire

$$T(s) = C_2 (sE_2 - A_2)^{-1} B_2 \quad ; \quad \begin{bmatrix} -1 & 0 \\ s & -1 \end{bmatrix}^{-1} = \frac{1}{1}\begin{bmatrix} -1 & 0 \\ -s & -1 \end{bmatrix}$$

$$= \begin{bmatrix} c_{1_2} & c_{2_2} \end{bmatrix}\begin{bmatrix} -1 & 0 \\ -s & -1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} c_{1_2} & c_{2_2} \end{bmatrix}\begin{bmatrix} -1 \\ -s \end{bmatrix} = \begin{bmatrix} 0 & -g_1 \end{bmatrix}\begin{bmatrix} -1 \\ -s \end{bmatrix}$$

$$= g_1 s$$

Return to

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -a_0 & -a_1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$y = [c_0 \;\; c_1] x \qquad T(s) = y(s) = \frac{y}{u}$$

$$= \frac{c_0 + c_1 s}{s^2 + a_1 s + a_0}$$

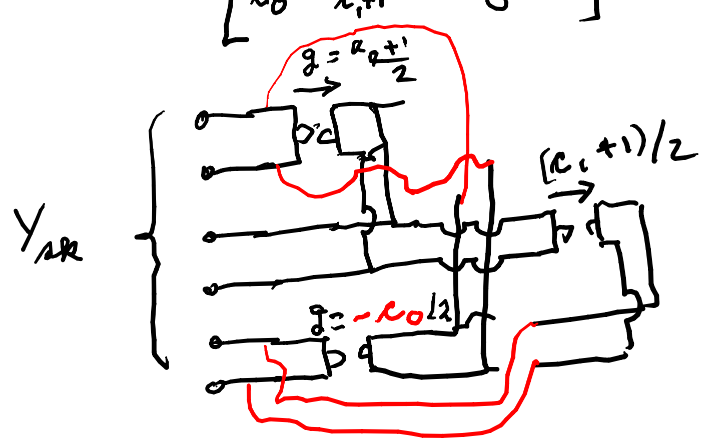
$$Y = \begin{bmatrix} -A & -B \\ C & 0 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ a_0 & a_1 & -1 \\ c_0 & c_1 & 0 \end{bmatrix}$$

can realize by VCCS each term separately

Energy for a gyrator $v^T i = [v_1 \; v_2]\begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$; $i = \begin{bmatrix} 0 & -g \\ g & 0 \end{bmatrix} v$

$$= [v_1 \; v_2]\begin{bmatrix} -g v_2 \\ g v_1 \end{bmatrix} = -g v_1 v_2 + g v_2 v_1 \equiv 0$$

$$2 Y_{sk} = \begin{bmatrix} 0 & -1-a_0 & -c_0 \\ a_0+1 & 0 & -(c_1+1) \\ c_0 & c_1+1 & 0 \end{bmatrix}$$



$$2Y_{sym} = \begin{bmatrix} 0 & a_0-1 & c_0 \\ a_0-1 & a_1 & c_1-1 \\ c_0 & c_1-1 & 0 \end{bmatrix};\quad \Delta_1 = \begin{vmatrix} 0 & a_0-1 \\ a_0-1 & a_1 \end{vmatrix} = -(a_0-1)^2$$

$\Rightarrow Y_{sym}$ is not > 0

1st put a transformation on the state
a similarity one

$$s 1_2 x = \begin{bmatrix} 0 & 1 \\ -a_0 & -a_1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u, \quad y = [c_0\ c_1] x$$

$$sTx = T\begin{bmatrix} 0 & 1 \\ -a_0 & -a_1 \end{bmatrix} T^{-1} \cdot Tx + T\begin{bmatrix} 0 \\ 1 \end{bmatrix} u, \quad y = [c_0\ c_1] T^{-1} Tx$$

$$y(s) = CT^{-1}(s1_2 - TAT^{-1})^{-1} TB)$$

$$= CT^{-1} \cdot T(s1_2 - A)^{-1} T^{-1} TB = C(s1_2 - A)^{-1} B$$

$$Y_T = \begin{bmatrix} -TAT^{-1} & -TB \\ CT^{-1} & 0 \end{bmatrix} = \begin{bmatrix} T & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -A & -B \\ C & 0 \end{bmatrix} \begin{bmatrix} T^{-1} & 0 \\ 0 & 1 \end{bmatrix}$$

$$Y_{T_{sym}} = \begin{bmatrix} -TAT^{-1} - T^{-T}A^T T^T & -TB + T^{-T}C^T \\ CT^{-1} - B^T T^T & 0 \end{bmatrix}$$

form
$$\begin{bmatrix} T^T & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -TAT^{-1} - T^{-T}A^T T^T & -TB + T^{-T}C^T \\ CT^{-1} - B^T T^T & 0 \end{bmatrix} \begin{bmatrix} T & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -T^T TA - A^T T^T T & -T^T T B + C \\ C - B^T T^T T & 0 \end{bmatrix} = \begin{bmatrix} -QA - A^T Q & -QB + C \\ C - B^T Q & 0 \end{bmatrix}$$

$$C - B^T Q = [c_0\ c_1] - [0\ 1] \begin{bmatrix} q_{11} & q_{12} \\ q_{12} & q_{22} \end{bmatrix} = [0, 0]$$

$$c_0 = +q_{12}, \quad c_1 = +q_{22}$$