EE610

10/09/12

Passive via state space

$$y(s) = \frac{i(s)}{v(s)} = \frac{c_1 s + c_0}{s^2 + a_1 s + a_0}$$

Passivity ⇒ $y(s)$ is PR

$$\frac{1}{y(s)} \Rightarrow \begin{array}{r|l} & (1/c_1)s \\ \hline c_1 s + c_0 & s^2 + a_1 s + a_0 \\ & s^2 + \frac{c_0}{c_1} s \\ \hline & (a_1 - \frac{c_0}{c_1}) s + a_0 \end{array} \Rightarrow \frac{1}{y(s)} = \frac{1}{c_1} s + \frac{(a_1 - \frac{c_0}{c_1}) s + a_0}{c_1 s + c_0}$$

need $c_1 > 0$ as otherwise $\frac{1}{y}$ has a double pole at ∞.

$$\frac{(a_1 - \frac{c_0}{c_1}) s}{c_1 s + c_0} + \frac{a_0}{c_1 s + c_0} \Rightarrow PR \Rightarrow \begin{array}{ll} a_1 - c_0/c_1 \geq 0, & a_0 \geq 0 \\ c_1 > 0 & c_0 \geq 0 \end{array}$$

Create state-variable equations

$$s 1_2 \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -a_0 & -a_1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} v \qquad x = \text{state, 2-D} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$i = [c_0 \ \ c_1] x$$

Let $P = 2\times2$ matrix, $\det P \neq 0$, $PP^{-1} = 1_2$

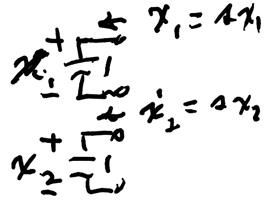
$$s P x = P \begin{bmatrix} 0 & 1 \\ -a_0 & -a_1 \end{bmatrix} P^{-1} P x + P \begin{bmatrix} 0 \\ 1 \end{bmatrix} v \qquad \Rightarrow \hat{x} = P x$$

$$i = [c_0 \ \ c_1] P^{-1} P x$$

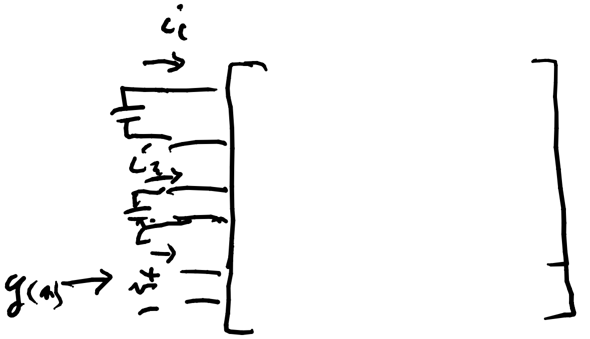
$$s 1_2 \hat{x} = \hat{A} \hat{x} + \hat{B} v, \quad i = \hat{C} \hat{x}; \quad \hat{A} = P \begin{bmatrix} 0 & 1 \\ -a_0 & -a_1 \end{bmatrix} P^{-1}$$ similarity transformati

$$\hat{B} = PB, \quad \hat{C} = CP^{-1}$$

Go to coupling Y_c matrix, 3×3



$$\begin{array}{r} -s x_1 = \\ -s x_2 = \\ \\ \end{array} \begin{bmatrix} i_1 \\ i_2 \\ \hat{i} \end{bmatrix} = \left[\begin{array}{c|c} -A & -B \\ \hline C & 0 \end{array}\right] \begin{bmatrix} x_1 \\ x_2 \\ v \end{bmatrix}; \quad Y_c = \left[\begin{array}{cc|c} 0 & -1 & 0 \\ a_0 & a_1 & -1 \\ \hline c_0 & c_1 & 0 \end{array}\right]$$



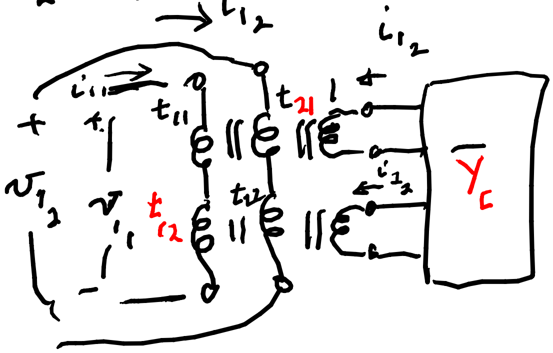
$$\hat{Y}_c = (P \dotplus 1)\, Y_c\, (P^{-1} \dotplus 1)$$

$$Y_c = Y_{c\,sym} + Y_{c\,skew} = Y_{c\,sym}^T - Y_{c\,skew}^T$$
$$= \frac{Y_c + Y_c^T}{2} + \frac{Y_c - Y_c^T}{2}$$

$$Y_{sym} = \frac{Y_c + Y_c^T}{2} = \frac{\begin{bmatrix} 0 & -1 & 0 \\ a_0 & a_1 & -1 \\ c_0 & c_1 & 0 \end{bmatrix} + \begin{bmatrix} 0 & a_0 & c_0 \\ -1 & a_1 & c_1 \\ 0 & -1 & 0 \end{bmatrix}}{2} = \frac{1}{2}\begin{bmatrix} 0 & a_0-1 & c_0 \\ a_0-1 & 2a_1 & c_1-1 \\ c_0 & c_1-1 & 0 \end{bmatrix}$$

$$\frac{Y_c - Y_c^T}{2} = \frac{\begin{bmatrix} 0 & -1 & 0 \\ a_0 & a_1 & -1 \\ c_0 & c_1 & 0 \end{bmatrix} - \begin{bmatrix} 0 & a_0 & c_0 \\ -1 & a_1 & c_1 \\ 0 & -1 & 0 \end{bmatrix}}{2} = \begin{bmatrix} 0 & -(a_0+1)/2 & -c_0/2 \\ \frac{a_0+1}{2} & 0 & -(c_1+1)/2 \\ +c_0/2 & \frac{c_1+1}{2} & 0 \end{bmatrix}$$

$$\begin{bmatrix} P & 0 \\ 0 & 1 \end{bmatrix} Y_{sym} \begin{bmatrix} P^{-1} & 0 \\ 0 & 1 \end{bmatrix} = \hat{Y}_{sym}$$



$$v_1 = \begin{bmatrix} v_{1_1} \\ v_{1_2} \end{bmatrix}, \quad v_2 = \begin{bmatrix} v_{1_2} \\ v_{2_2} \end{bmatrix}$$

i_1, i_2

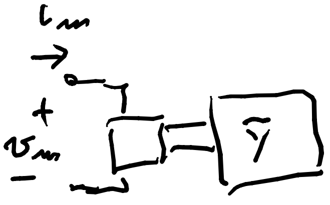
$$v_1 = T^T v_2 = \begin{bmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{bmatrix} v_2 \; ; \quad v_1^T i_1 + v_2^T i_2 = 0$$

$$v_2^T T^T i_1 = -v_2^T i_2$$

$$i_2 = -T^T i_1 \implies i_{in} = T\hat{i}$$

$$\hat{i} = \bar{Y}_c \hat{v}$$

$$i_{in} = T \bar{Y}_c T^T v_{in}$$



$$\implies Y = T \bar{Y} T^T$$

congruency transformation

note $(P^{-1}P^{-T})^T = P^{-1}P^{-T}$

$$\underbrace{(P^{-1} \dotplus 1)(P \dotplus 1)}_{1_3} \begin{bmatrix} 0 & -1 & 0 \\ a_0 & a_1 & 1 \\ c_0 & c_1 & 0 \end{bmatrix} \underbrace{(P^{-1} \dotplus 1)(P^{-T} + 1)}_{\text{symmetric positive definite}}$$

$$\begin{bmatrix} 0 & -1 & 0 \\ a_0 & a_1 & 1 \\ c_0 & c_1 & 0 \end{bmatrix}\begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -A & B \\ C & 0 \end{bmatrix}\begin{bmatrix} Q & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -AQ & B \\ CQ & 0 \end{bmatrix}$$

desire symmetric part to be positive semidefinite

symmetric part of the above 3 port admittance

$$\begin{bmatrix} -AQ - Q^TA^T & B - Q^TC^T \\ CQ - B^T & 0 \end{bmatrix};$$

$$CQ - B^T = [c_0 \; c_1]\begin{bmatrix} Q_{11} & Q_{12} \\ Q_{12} & Q_{22} \end{bmatrix} - [0 \;\; 1]$$

$$c_0 Q_{11} + c_1 Q_{12} = 0$$

$$c_0 Q_{12} + c_1 Q_{22} = 1$$

$$Q_{12} = -\frac{c_0}{c_1} Q_{11}$$

$$Q_{22} = \frac{1 - c_0\left(-\frac{c_0}{c_1}\right) Q_{11}}{c_1}$$

need $CQ - B^T = [0 \;\; 0]$ for the symmetric part to be positive semi-definite

$$-AQ - Q^TA^T$$

$$-\begin{bmatrix} 0 & +1 \\ -a_0 & -a_1 \end{bmatrix}\begin{bmatrix} Q_{11} & Q_{12} \\ Q_{12} & Q_{22} \end{bmatrix} - \begin{bmatrix} Q_{11} & Q_{12} \\ Q_{12} & Q_{22} \end{bmatrix}\begin{bmatrix} 0 & -a_0 \\ 1 & -a_1 \end{bmatrix} = \Pi$$

(1,1) $-Q_{12} - Q_{12} = -2Q_{12} = 2\frac{c_0}{c_1} Q_{11} = \Pi_{11}$

(1,2) $-Q_{22} + (a_0 Q_{11} + a_1 Q_{12}) = -\frac{1}{c_1} - \frac{c_0^2}{c_1^2} Q_{11} + a_0 Q_{11} - a_1 \frac{c_0}{c_1} Q_{11} = -\frac{1}{c_1} + (a_0 - a_1\frac{c_0}{c_1} - \frac{c_0^2}{c_1}$

$= (2,1)$ as symmetric $\qquad = \Pi_{12} = \Pi_{21}$

(1,2) $a_0 Q_{12} + a_1 Q_{22} + a_0 Q_{12} + a_1 Q_{22} = 2a_0 Q_{12} + 2a_1 Q_{22} = \Pi_{22}$

$$= -2a_0\frac{c_0}{c_1} Q_{11} + \frac{2a_1}{c_1} + 2a_1\left(\frac{c_0}{c_1}\right)^2 Q_{11} = \frac{2a_1}{c_1} + 2\left(a_1\left(\frac{c_0}{c_1}\right)^2 - a_0\frac{c_0}{c_1}\right) Q_{11}$$

due to PR condition Π is positive semi-definite

$\Rightarrow$ can make with transformers & positive resistors