EE 610

10/02/12

Note Title 10/2/2012

PR Lossless $\quad y(s) + y(-s) = 0$

$$y(s) = \frac{k_0}{s} + k_\infty s + \sum \frac{k_i}{s - j\omega_i} + \frac{k_i}{s + j\omega_i} \quad ; \; k_i > 0 \text{ (real)}$$

$$= \frac{k_0}{s} + k_\infty s + \sum_{i=1}^{k} \frac{2k_i s}{s^2 + \omega_i^2}$$

$\delta[f(s)]$ = degree = highest power of s in numerator or denominator of a rational $f(s) = \frac{N(s)}{D(s)}$

N & D polynomial

for above, if $k_\infty = 0$; $\delta[y(s)] = 2k + 1$

if $k_0 \neq 0$ $\quad$ if $k_\infty \neq 0$ $\quad \delta[y(s)] = 2k + 2$

Here can synthesize with $\delta[y(s)]$ = # of L's & C's

$+ \Rightarrow$ parallel

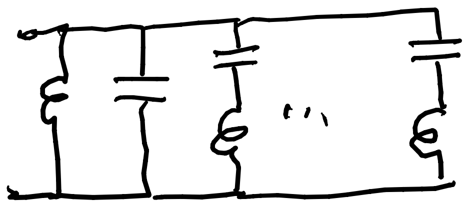
$\frac{k_0}{s} = y_0(s) =$  $L_0 = 1/k_0$

$k_\infty \cdot s = y_\infty(s) =$  $C_\infty = k_\infty$

$$\frac{2k_i s}{s^2 + \omega_i^2} = \frac{1}{\frac{s^2}{2sk_i} + \frac{\omega_i^2}{2k_i s}} = \frac{1}{\frac{s}{2k_i} + \frac{1}{\frac{2k_i}{\omega_i^2} s}} = y_i(s)$$

$$\frac{1}{y_i} = z_i = \frac{1}{2k_i} s + \frac{1}{\frac{2k_i}{\omega_i^2} s}$$

 $C_i = \frac{2k_i}{\omega_i^2}$, $L_i = \frac{1}{2k_i}$

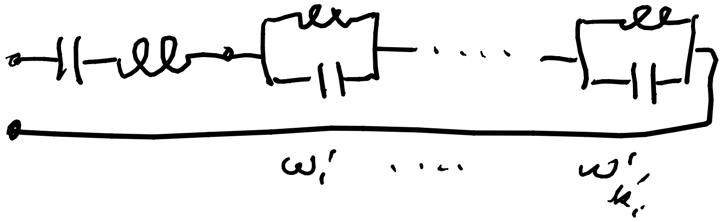
$y(s) =$ 

uses $\delta[y]$ reactive elements $\quad \left(\omega_i = \frac{1}{\sqrt{L_i C_i}}\right)$

(= minimum number possible ⇒ canonical form)

= 2nd Foster form

The 1st is the dual, from $z(s)$ or by "dual circuit"

$y(s)$ 
$\leftarrow \omega_i' \Rightarrow$ zeros of y
$=$ poles of z

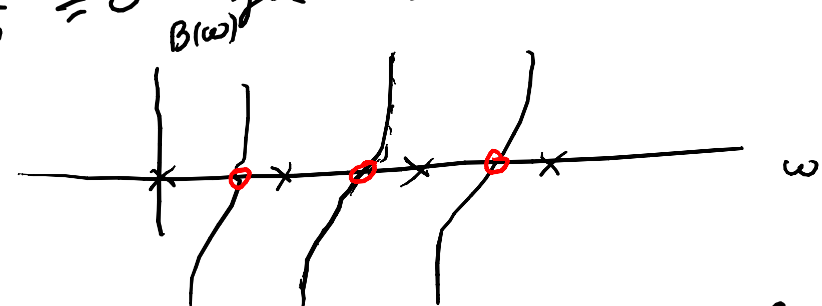
1st observe zeroes & poles of $y(s)$ [lossless PR] alternate

$$y(j\omega) = \frac{k_0}{j\omega} + k_\infty j\omega + \sum_{i=1}^{k} \frac{2k_i j\omega}{-\omega^2+\omega_i^2} = jB(\omega)$$

$$\frac{dB(\omega)}{d\omega} = \frac{k_0}{\omega^2} + k_\infty + \sum_{i=1}^{k}\left(\frac{2k_i}{-\omega^2+\omega_i^2} - \frac{2k_i\omega(-2\omega)}{(-\omega^2+\omega_i^2)^2}\right)$$

$$= \frac{k_0}{\omega^2} + k_\infty + \sum_{i=1}^{k} \underbrace{\frac{1}{(-\omega^2+\omega_i^2)^2}\left[-2k_i\omega^2 + 2k_i\omega_i^2 + \cancel{4}\,2\,k_i\omega^2\right]}_{\geq 0}$$

$$\therefore \frac{dB(\omega)}{d\omega} \geq 0 \quad \text{for } -\infty < \omega < \infty$$



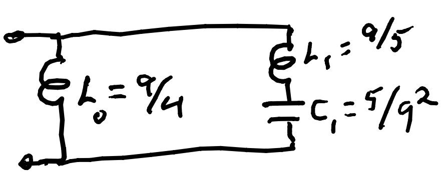
shows that poles & zeroes alternate on the $j\omega$ axis for a lossless PR function.

$$y(s) = \frac{(s^2+4)}{s(s^2+9)} \quad \text{is a lossless PR function}$$

$$= \frac{k_0}{s} + \frac{2k_1 s}{s^2+9}; \quad k_0 = y(s)\times s\Big|_{s=0} = \frac{s^2+4}{s^2+9}\Big|_{s=0} = 4/9$$

$$= \frac{4/9}{s} + \frac{5/9\, s}{s^2+9} \qquad 2k_1 = y(s)\frac{(s^2+9)}{s}\Big|_{s^2=-9} = \frac{s^2+4}{s^2}\Big|_{s^2=-9} = \frac{-5}{-9} = 5/9$$

2nd Foster

$$\frac{s^2+4}{s(s^2+9)} = y(s) \rightarrow$$ 

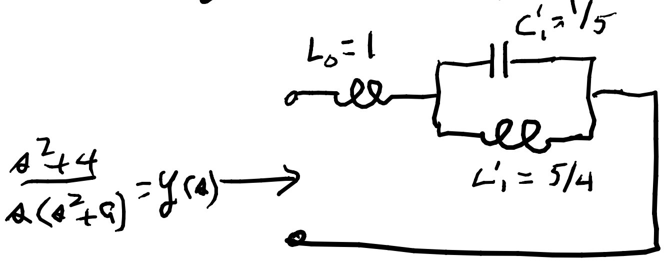
$$\frac{5/9\, s}{s^2+9} = \frac{1}{\frac{s}{5/9} + \frac{9}{5/9\, s}} = z_1$$

1st Foster $Z(s) = \frac{1}{Y(s)} = \frac{s(s^2+9)}{s^2+4} = 1\cdot s + \frac{2k_1' s}{s^2+4}$

$$2k_1' = \left[\frac{s^2+4}{s} \times \left(\frac{s(s^2+9)}{s^2+4}\right) - 1\cdot s\left(\frac{s^2+4}{s}\right)\right]\Bigg|_{s^2=-4}$$

$$= s^2+9\Big|_{s^2=-4} + 0 = +5$$

$$Z(s) = \frac{1}{Y(s)} = s + \frac{5s}{s^2+4}$$

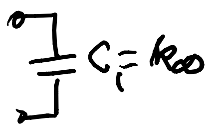


$$Z_1 = \frac{5s}{s^2+4} = \frac{1}{\frac{s}{5}+\frac{4}{5s}}$$

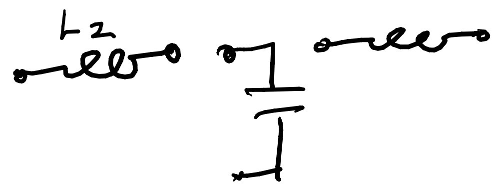
1st Cauer removes poles @ ∞ in a continued fraction expansion

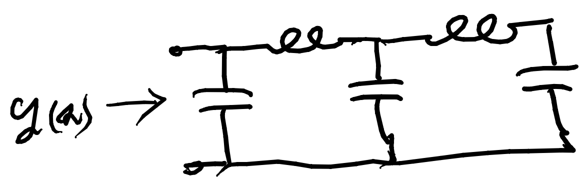
2nd Cauer " " @ 0 " " "

1st Cauer: $Y(s)$ is there a pole at ∞,



then $\frac{1}{\text{remainder}}$ also has a pole at ∞





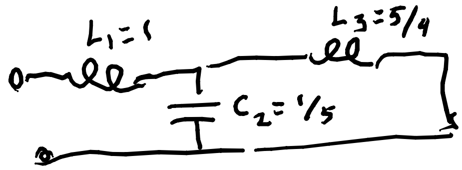
canonical gives low-pass 2-ports

Ex: $Y(s) = \frac{s^2+4}{s(s^2+9)} = \frac{s^2+4}{s^3+9s} = \frac{1}{\frac{s^3+9s}{s^2+4}} = \frac{1}{s+\frac{5s}{s^2+4}}$

$$= \frac{1}{s + \frac{1}{\frac{1}{5}s + \frac{1}{\frac{5s}{4}}}}$$

$$\begin{array}{r|l} & s \\ s^2+4 & s^3+9s \\ & s^3+4s \\ \hline & 5s \end{array} \qquad \begin{array}{r|l} & \frac{1}{5}s \\ 5s & s^2+4 \\ & s^2 \\ \hline & 4 \end{array}$$

1st Cauer



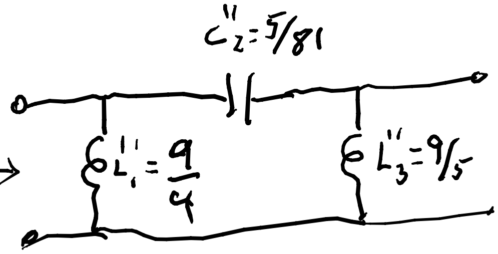
2nd Cauer $\quad y(s)=\dfrac{4+s^2}{9s+s^3}=\dfrac{4}{9s}+\cfrac{1}{\dfrac{81}{5s}+\cfrac{1}{\dfrac{5}{9s}+0}}$ $\quad \leftarrow y \quad \leftarrow z \quad \leftarrow y$

$$
\begin{array}{r|l}
 & \dfrac{4}{9s} \\
9s+s^3 & 4+s^2 \\
 & 4+\dfrac{4}{9}s^2 \\
\hline
\end{array}
$$

$$
\begin{array}{r|l}
 & \dfrac{9^2}{5s} \\
\dfrac{5}{9}s^2 & 9s+s^3 \\
 & 9s+ \\
\hline
\end{array}
$$

$$
\begin{array}{r|l}
 & 5/9s \\
s^3 & \dfrac{5}{9}s^2 \\
\end{array}
$$

$\dfrac{s^2+4}{s(s^2+9)}=y(s)\rightarrow$



(gives high pass 2-ports)

all 4 methods use the minimum number of L, C, s.