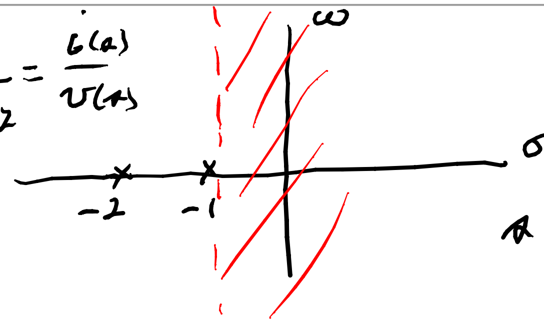
EE 610

09/27/12

Note Title 9/25/2012

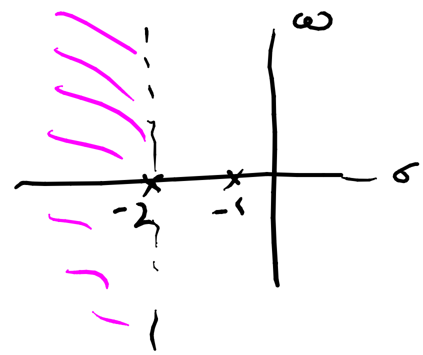
$$y(s) = \frac{1}{s+1} + \frac{2}{s+2} = \frac{(3s+4)}{s^2+3s+2} = \frac{i(s)}{v(s)}$$



s - plane

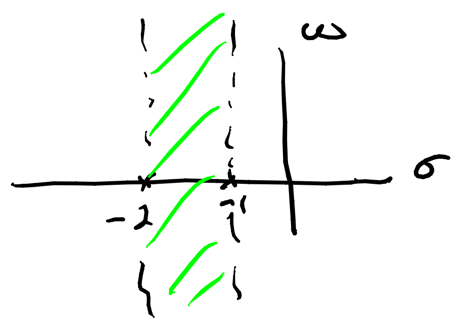
$$\mathcal{L}^{-1}[y(s)] = e^{-t}\,1(t) + 2e^{-2t}\,1(t)$$

if $\sigma > -1$ (causal)



$$\mathcal{L}^{-1}[y(s)] = -e^{-t}\,1(-t) - 2e^{-2t}\,1(-t)$$

if $\sigma < -2$ (non causal)



$$\mathcal{L}^{-1}[y(s)] = -e^{-t}\,1(-t) + 2e^{-2t}\,1(t)$$

if $-2 < \sigma < -1$ (non causal)

$$(s^2+3s+2)\,i(t) = (3s+4)\,v(t)$$

$$\frac{d^2 i(t)}{dt^2} + 3\frac{di(t)}{dt} + 2i(t) = 3\frac{dv(t)}{dt} + 4v(t).$$

has 3 solutions for $v(t) = \delta(t)$

PR functions = rational in s and positive real

($\Leftrightarrow$ physically realizable, $R \geq 0$, $L \geq 0$, $C \geq 0$, transformer, gyrators)

condions for PR $f(s)$

1. real coefficients, $f(\sigma)$ is real for real $s = \sigma$
2. no poles in $\sigma > 0$; $f(s)$ is analytic in $\sigma > 0$
3. $\mathrm{Re}\, f(s) \geq 0$ in $\sigma > 0$

If $f(s) = y(s)$ an admittance is PR, then $f(s) = z(s)$ an impedance is PR; $\frac{1}{f(s)}$ is PR

$$\left(\frac{1}{f(s)} + \frac{1}{f^*(s)}\right) = 2\,\mathrm{Re}\left(\frac{1}{f(s)}\right) = \frac{f(s)+f^*(s)}{f(s)f^*(s)} = \frac{2\,\mathrm{Re}\, f(s)}{|f(s)|^2}$$

$\Rightarrow \mathrm{Re}\, f(s) > 0 \Rightarrow \mathrm{Re}\left(1/f(s)\right) > 0$

Ex: $y_L(s) = \frac{1}{Ls} \Rightarrow y_C(s) = Cs = \frac{1}{1/Ls} \Rightarrow L = C$

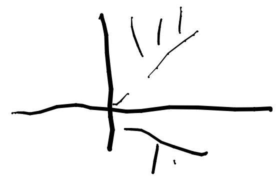
check: $f(s) = s = \sigma + j\omega$, $\mathrm{Re}\, s = \sigma > 0$ in $\sigma > 0$

$\therefore f(s) = s$ is PR

$\Rightarrow \frac{1}{s}$ is PR

Ex: $y(s) = s^2 = (\sigma + j\omega)^2 = \sigma^2 - \omega^2 + 2j\omega\sigma$

$\mathrm{Re}\, y(s) = \sigma^2 - \omega^2$ can be < 0 in $\sigma > 0$ if $\omega > \sigma$

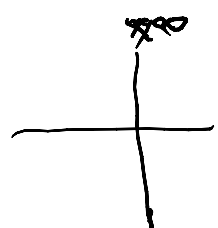


$\therefore y(s) = s^2$ is not PR

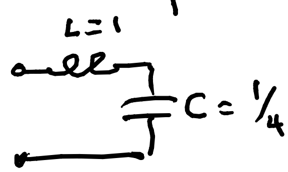
$\Rightarrow y(s) = \frac{1}{s^2}$ is not PR

Ex $y(s) = \frac{1}{s}$

; $y = s$



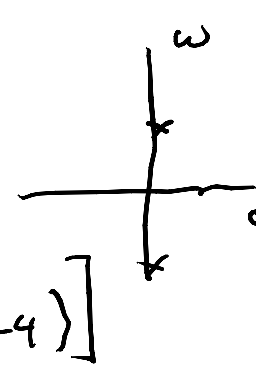
Ex: $y(s) = \frac{s}{s^2+4} = \frac{1}{s + \frac{4}{s}} \Rightarrow$



1. ok, rational with real coefficients

2. ok, no singularity (poles) in $\sigma > 0$

one @ $s^2 + 4 = 0$; $s = \pm j2$



3. $\mathrm{Re}(y(s))\big|_{\sigma>0} = \mathrm{Re}\left[(\sigma + j\omega) / ((\sigma+j\omega)^2 + 4)\right]$

den: $(\sigma^2-\omega^2+4+j2\sigma\omega)$

$$\mathrm{Re}\left[\frac{(\sigma+j\omega)(\sigma^2-\omega^2+4-j2\sigma\omega)}{(\sigma^2-\omega^2+4)^2+(2\sigma\omega)^2}\right] \Rightarrow \text{check } \left(\sigma(\sigma^2-\omega^2+4)+2\omega^2\sigma\right)$$

$$= \sigma^2+4\sigma+\omega^2\sigma$$

> 0 in $\sigma > 0$

$\Rightarrow \dfrac{s}{s^2+4}$ is PR

$\Rightarrow$ can have poles & zeros on $j\omega$ axis

Ex: Lossless PR functions

$P_{ave}(j\omega) = 0$ in sinusoidal steady state

$$= \mathrm{Re}(V^*I) = \frac{V^*I + VI^*}{2}$$

if $y(s)$ = admittance $\Rightarrow 2P_{ave}(j\omega) = V^*y(j\omega)V + V y^*(j\omega) V^*$

$$= V^*(y(j\omega)+y^*(j\omega))V$$

$\Rightarrow y(j\omega)+y^*(j\omega) = 0$ for all ω ($\neq$ a pole)

$2\,\mathrm{Ev}(y(j\omega)) = y(j\omega)+y(-j\omega) = 0 \Rightarrow (y(s)+y(-s))\Big|_{s=j\omega} = 0$

$\Rightarrow$ by analytic continuation

$y(s)+y(-s) = 0$ for all s

Ex: $\dfrac{s}{s^2+4} = y(s)$ $\quad y(-s) = \dfrac{-s}{s^2+4}$; $\quad y(s)+y(-s) = \dfrac{s-s}{s^2+4} = 0$

$\Rightarrow \mathrm{Ev}\, y(s) \equiv 0$ for a lossless PR $y(s)$ (lossless 1-ports)

$\Rightarrow$ all poles are on $j\omega$ axis

as $\Rightarrow$

$\Rightarrow$ all zeros are on $j\omega$ axis

$\times \leftarrow$ if in $y(-s) = -y(s)$

pole of $y(s)$ in $\sigma > 0$ if one in $\sigma < 0$

$y(s)$ $\times$

These poles are simple, as are any poles of a positive real function, with a real positive residue

Ex: $Y(s) = \frac{s}{s^2+4} = \frac{k_1}{s+j2} + \frac{k_2}{s-j2}$

$$k_1 = (s+j2)\left[\frac{s}{s^2+4} - \frac{k_2}{s-j2}\right]\Bigg|_{s=-j2} = \frac{s}{s-j2}\Bigg|_{s=-j2} + \cancel{\frac{k_2(s+j2)}{s-j2}\Bigg|_{s=-j2}}$$

$$= \frac{-j2}{-j4} = 1/2$$

$$\Rightarrow k_2 = 1/2 = k_1^*$$

Reason for simple poles & $k > 0$

$$\approx F(s) \approx \frac{k}{(s-j\omega_0)^n} = \frac{k}{\rho^n e^{jn\angle(s-j\omega_0)}} = \frac{|k|}{\rho^n} e^{j\angle k - n\angle(s-j\omega_0)}$$

$\angle$ can go from $-\pi/2$ to $+\pi/2$ for $\sigma > 0$

need this Re $\geq 0 \Rightarrow \cos(\angle k - n\angle(s-j\omega_0))$ need ≥ 0

true only if $n=1$ & $\angle k = 0 \Rightarrow k$ is real & the pole is simple