EE 610

09/20/12

Note Title 9/20/2012

Richards' function, p. 361

$$R(s) = \frac{k z(s) - s z(k)}{k z(k) - s z(s)} \Rightarrow \frac{1}{R(s)} = G(s) = \frac{k y(k) - s y(s)}{k y(s) - s y(k)}$$

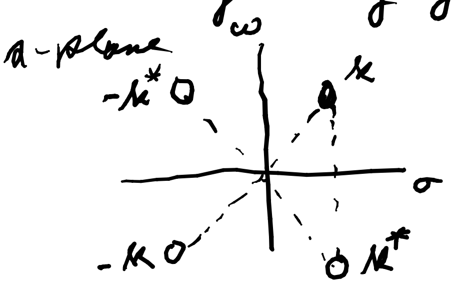
$$G(k) = \frac{k y(k) - k y(k)}{k y(k) - k y(k)} = \frac{0}{0} \Rightarrow s-k \text{ factors numerator \& denominator if } y(s) \text{ is rational in } s$$

$$G(-k) = \frac{k y(k) - (-k) y(-k)}{k y(-k) - (-k) y(k)} = \frac{k y(k) + k y(-k)}{k y(-k) + k y(k)} = \frac{k}{k} \frac{(y(k) + y(-k))}{(y(k) + y(-k))}$$

$$= \frac{0}{0} \text{ if } y(k) = -y(-k) \Rightarrow \text{if } \mathrm{Ev}\, y(s) = \tfrac{1}{2}(y(s) + y(-s))$$

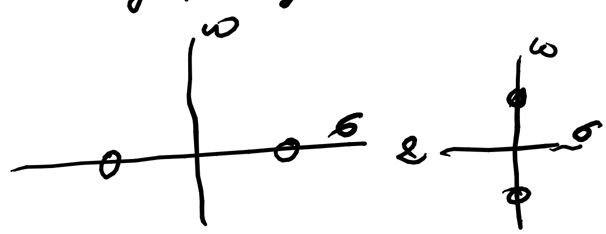
has a zero at $s = k$

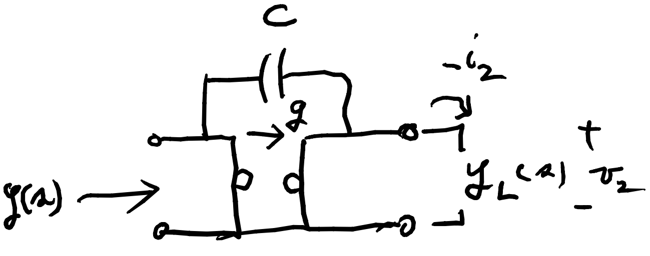
allows degree of $G(s)$ to go down by 1 from the degree of $y(s)$, if $y(s)$ is a ratio of polynomials




$\mathrm{Ev}\, y(s)$ for rational $y(s)$ with real coefficients

$s = \sigma + j\omega$




$$Y = \begin{bmatrix} Cs & -Cs - g \\ -Cs + g & Cs \end{bmatrix} ; \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} Cs & -Cs - g \\ -Cs + g & Cs \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$$i_2 = -y_L v_2$$

$$\Rightarrow (-y_L - Cs) v_2 = (-Cs + g) v_1$$

$$i_1 = y \cdot v_1 = \left[ Cs + \frac{(-Cs - g)(-Cs + g)}{-y_L - Cs} \right] v_1$$

$$y(s) = y_{11} + y_{12} \frac{1}{-y_L - y_{22}} \times y_{21} = \frac{-y_L y_{11} - y_{11} y_{22} + y_{12} y_{21}}{-y_L - y_{22}}$$

$$= (y_L y_{11} + \Delta y) / (y_L + y_{22})$$

$$= (sC\,y_L + g^2)/(y_L + sC) \Rightarrow \quad y \cdot y_L + y \cdot sC = sC\,y_L + g^2$$

$$(y - sC)\,y_L = g^2 - sC \cdot y$$

$$y_L(s) = \frac{g^2 - sC \cdot y(s)}{y(s) - sC} \qquad \text{compare} \qquad \frac{k\,y(k) - s\,y(s)}{k\,y(s) - s\,y(k)}$$

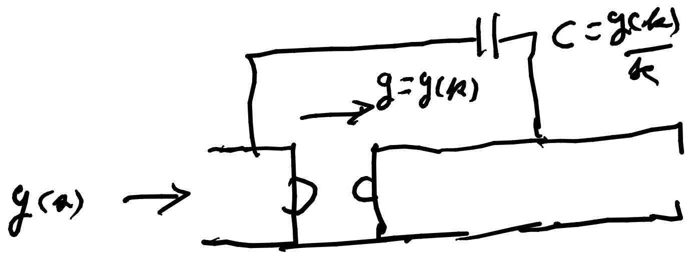
$$= g^2 \frac{\left(1 - \frac{sC}{g} \cdot \frac{y(s)}{g}\right)}{y(s) - sC} \qquad \frac{k\,y(k)\left[1 - \frac{s}{k} \cdot \frac{y(s)}{y(k)}\right]}{k\,y(k)\left[\frac{y(s)}{y(k)} - \frac{s}{k}\right]}$$

$$\frac{y_L(s)}{g} = \frac{1 - \frac{sC}{g} \cdot \frac{y(s)}{g}}{\frac{y(s)}{g} - \frac{sC}{g}} \Longrightarrow \quad = \frac{1 - \frac{s}{k} \cdot \frac{y(s)}{y(k)}}{\frac{y(s)}{y(k)} - \frac{s}{k}}$$

take

$$\Longrightarrow k = \frac{g}{C}, \qquad g = y(k)$$

$$\Longrightarrow C = \frac{g}{k} = \frac{y(k)}{k}$$




$$y_L = g\,G(s) = y(k) \cdot \frac{k\,y(k) - s\,y(s)}{k\,y(s) - s\,y(k)}$$

allows synthesis if $k$ is real

look at lossless circuits:

$p(t) \Rightarrow$ average power in sinusoidal steady state

$P(\omega) = 0$ if lossless $\quad \mathrm{Re}(V^{*T} I) = 0 = \mathrm{Re}[V_1^* I_1 + V_2^* I_2 \cdots V_m^* I_m]$

$$\mathrm{Re}\, c = \tfrac{1}{2}(c + c^*) = \tfrac{1}{2}(c + c^{*T}) \qquad j^* = -\sqrt{-1} = -j$$

$$\mathrm{Re}\, V^{*T} I = \tfrac{1}{2}(V^{*T} I + I^{*T} V) = 0 \quad \text{if } I = Y(j\omega) V$$

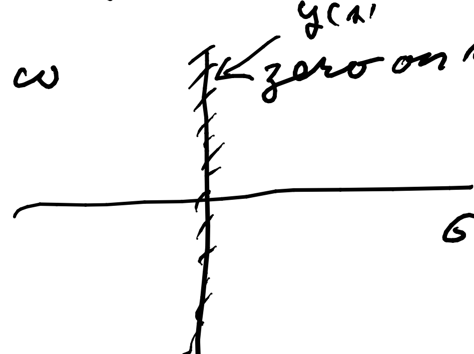
$$= \tfrac{1}{2}(V^{*T} Y(j\omega) V + V^{*T} Y^{*T}(j\omega) V) = \tfrac{1}{2} V^{*T}[Y(j\omega) + Y^{T*}(j\omega)] V$$

$$\frac{1}{2}\left[Y(j\omega) + Y^{T*}(j\omega)\right] = O_{m\times m}$$

$$= \frac{1}{2}\left[Y(j\omega) + Y^{T}(-j\omega)\right] = O_{m\times n}$$ if entries in $Y(s)$ are rational with real coefficients

for scalar $y(s)$ rational real coeffints

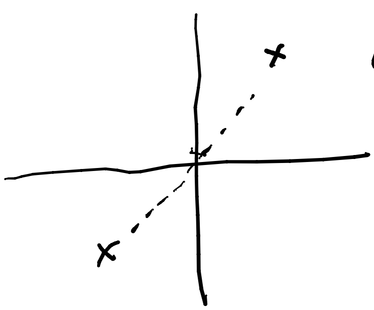
$$\frac{1}{2}\left(y(j\omega) + y(-j\omega)\right) = 0 \quad \text{for all } \omega$$




$y(s) + y(-s) \equiv 0$ for a lossless $y(s)$

since it is 0 on a dense set, $\omega$ axis.

$y(s) = -y(s) \Rightarrow \mathcal{E}v\, y(s) \equiv 0$ ∴ for the Richards' function can choose any $k$ we want



can't be if the circuit for $y(s)$ is stable $\frac{k_0}{s - s_0} \Rightarrow e^{s_0 t}$


⇓

all poles are on $j\omega$ axis

as $z(s) = \frac{1}{y(s)}$ satisfies $P(\omega) = 0 \Rightarrow$ poles of $z(s)$ are on the $j\omega$ axis. $\Rightarrow$ zeros of $y(s)$ are on $j\omega$ axis

see p. 342 no double poles $\longrightarrow \frac{k_0 s}{(s - s_0)^2} \longrightarrow$


$$y(s) = \frac{\prod_i (s - z_i)(s - z_i^*)}{\prod_k (s - p_k)(s - p_k^*)}$$

$$= \sum_k \frac{K_k}{s - j\omega_k} + \frac{K_k^*}{s + j\omega_k} + \quad \Rightarrow \text{need real residues } K_k = K_k^*$$

$$= \sum \frac{2K_k s}{s^2 + \omega_k^2} + K_\infty s \quad \text{is passive if } K_k > 0$$

poles & zeros will alternate

Ex: $$y(s) = \frac{2s}{s^2+1} + \frac{3s}{s^2+2} + \frac{1}{s} + 5s = \frac{a_6 s^6 + a_4 s^4 + a_2 s^2 + a_0}{s(b_4 s^4 + b_2 s^2 + b_0)}$$

$$= \frac{\text{Ev polynomial}}{\text{Od polynomial}}$$