EE610
09/18/12

Note Title 9/18/2012

SVD = singular value decomposition

M $n \times m$ real form MM^T $n \times n$

vector x form $x^T M \cdot \underbrace{M^T x}$ is 1×1

$$[y_1, \ldots, y_m] \begin{bmatrix} y_1 \\ \vdots \\ y_m \end{bmatrix} = \sum y_i^2 \geq 0$$

there exists a V such that $VM \cdot M^T V^T = \begin{bmatrix} D & 0 \\ 0 & 0 \end{bmatrix}$,

$VV^T = 1_m$, V is $n \times n$

D is diagonal and

$$D = \begin{bmatrix} d_1 & & & & \\ & \ddots & & & \\ & & d_n & & \\ & & & 0 & \\ & & & & \ddots 0 \end{bmatrix}; \quad d_i > 0$$

$$D^{1/2} = \begin{bmatrix} d_1^{1/2} & & & & \\ & \ddots & & & \\ & & d_n^{1/2} & & \\ & & & 0 & \\ & & & & \ddots 0 \end{bmatrix}$$

put U, $U \times U^T = 1_m$; $VMU_1 \cdot U_1^T M^T V^T = \begin{bmatrix} \sqrt{D} & \\ & 0 \end{bmatrix} \begin{bmatrix} \sqrt{D} & \\ & 0 \end{bmatrix}$

$$VMU_1 = \begin{bmatrix} \sqrt{D} & \\ & 0 \end{bmatrix} \Rightarrow VMU_1U_1^T = \begin{bmatrix} \sqrt{D} & 0 \\ 0 & 0 \end{bmatrix} U_1^T$$

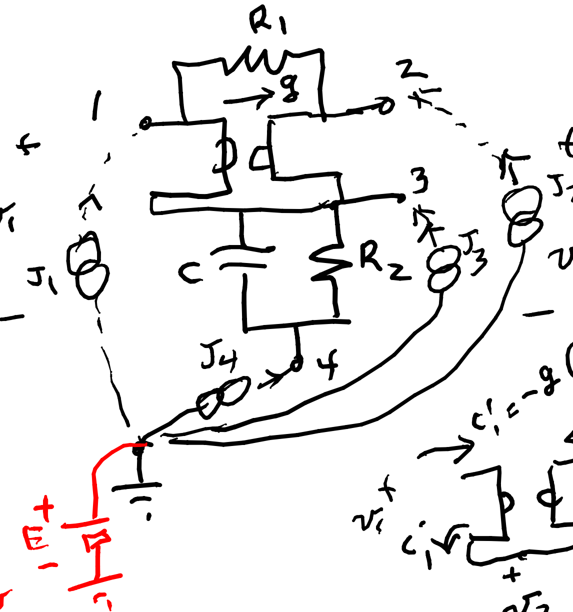
$$\Rightarrow VM = \begin{bmatrix} \sqrt{D} & 0 \\ 0 & 0 \end{bmatrix} U_1^T \Rightarrow U_1^T = \begin{bmatrix} \sqrt{D}^{-1} & 0 \\ 0 & 0 \end{bmatrix} VM$$

is singular due to 0 columns & rows

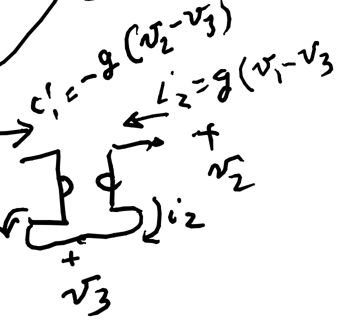
fill out columns of U_1 to make an orthogonal U

give $VMU = \begin{bmatrix} \sqrt{D} & 0 \\ 0 & 0 \end{bmatrix} \Rightarrow M = V^T \begin{bmatrix} \sqrt{D} & 0 \\ 0 & 0 \end{bmatrix} U^T$

Indefinite admittance $\Rightarrow$ rows sum to zero
columns sum to zero



$$\begin{bmatrix} J_1 \\ J_2 \\ J_3 \\ J_4 \end{bmatrix} = \underbrace{\begin{bmatrix} G_1 & -G_1-g & g & 0 \\ -G_1+g & G_1 & -g & 0 \\ -g & g & sC+G_2-g+g & -G_2-sC \\ 0 & 0 & -sC-G_2 & G_2+sC \end{bmatrix}}_{Y_{indef}} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix}$$

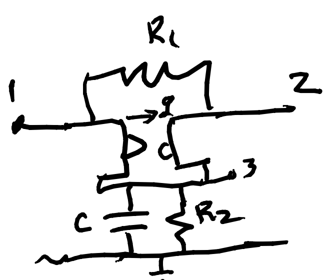


new ⇒ sum of entries on E will be zero since the J's are not changed and every v_i becomes v_i+E ⇒ entries on E are sums of columns

Now can move the ground onto the circuit; this forces one of the v_i's $=0$ ⇒ ignore the ith column as $\times 0$ & ignore the J_i'th current (as flows in ground) ⇒ scratch out i'th row & column ⇒ $Y_{definite}$

In above ground node 4, as an example, then

$$Y_{def} = \begin{bmatrix} G_1 & -G_1-g & g \\ -G_1+g & G_1 & -g \\ -g & g & sC+G_2 \end{bmatrix}$$



now look at if desire as a 2-port then to eliminate node 3

$$\begin{bmatrix} J_1 \\ J_2 \\ \hline J_3=0 \end{bmatrix} = \left[\begin{array}{c|c} Y_{11} & Y_{12} \\ \hline \underbrace{Y_{21}}_{[-g\ g]} & \underbrace{Y_{22}}_{[sC+G_2]} \end{array}\right] \begin{bmatrix} v_1 \\ v_2 \\ \hline v_3 \end{bmatrix} \Rightarrow 0 = Y_{21}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} + Y_{22} v_3$$

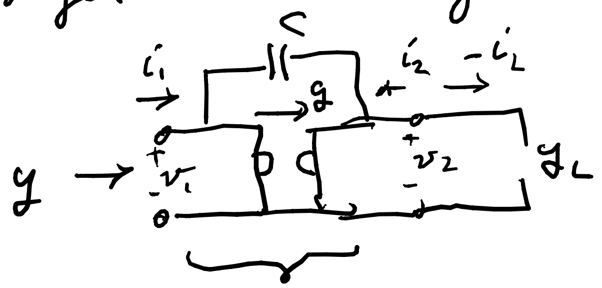
$$\Rightarrow v_3 = -Y_{22}^{-1} Y_{21} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$$\begin{bmatrix} J_1 \\ J_2 \end{bmatrix} = \left[Y_{11} + Y_{12}(-Y_{22}^{-1} Y_{21}) \right] \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = Y_{2\text{-port}} \cdot \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

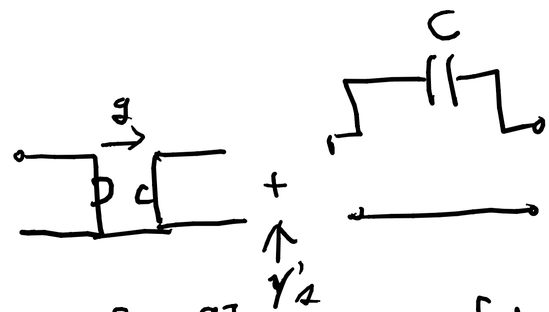
$$Y_{2\text{-port}} = \begin{bmatrix} G_1 & -G_1-g \\ -G_1+g & G_1 \end{bmatrix} - \begin{bmatrix} g \\ -g \end{bmatrix} \frac{1}{sC+G_2} \begin{bmatrix} -g & g \end{bmatrix}$$

$$= \begin{bmatrix} G_1 + \frac{g^2}{sC+G_2} & -G_1 - g - \frac{g^2}{sC+G_2} \\ -G_1 + g - \frac{g^2}{sC+G_2} & G_1 + \frac{g^2}{sC+G_2} \end{bmatrix} = G_1 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} + g \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} + \frac{g^2}{sC+G_2} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

To get Richards' function



$$Y_{coupling} = Y_g + Y_c$$



$$Y_g = \begin{bmatrix} 0 & -g \\ g & 0 \end{bmatrix} \qquad Y_c = sC \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} sC & -g-sC \\ g-sC & sC \end{bmatrix}$$

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} i_1 \\ -i_L \end{bmatrix} = \begin{bmatrix} i_1 \\ -y_L v_2 \end{bmatrix} = \begin{bmatrix} sC & -g-sC \\ g-sC & sC \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$$-y_L v_2 - sC v_2 = (g-sC) v_1 \Rightarrow v_2 = \frac{g-sC}{-(y_L+sC)} \cdot v_1$$

$$y = \frac{i_1}{v_1} = sC + \frac{(-g-sC)(g-sC)}{-(y_L+sC)} = \frac{y_L sC + \cancel{s^2C^2} + g^2 + \cancel{sCg} - \cancel{sCg} - \cancel{s^2C^2}}{y_L+sC}$$

$$y = \frac{g^2 + sC y_L}{sC + y_L} \Rightarrow y sC + y_L \cdot y = g^2 + sC y_L$$

$$y_L (y - sC) = g^2 - sC y$$

$$\Rightarrow y_L = \frac{g^2 - sCy}{y - sC}$$

Ev, Od

$$f(s) = \frac{f(s)+f(-s)}{2} + \frac{f(s)-f(-s)}{2}$$

$$= \text{Ev}\, f(s) + \text{Od}\, f(s)$$

$$\text{Ev}\, f(s) = \text{Ev}\, f(-s)$$
$$\text{Od}\, f(s) = -\text{Od}\, f(-s)$$

$y(s) = Cs$ is odd as $y(-s) = -Cs = -y(s) = -(Cs)$

$y(s) = \frac{1}{Ls}$ is odd as $y(-s) = \frac{1}{-Ls} = -\frac{1}{Ls} = -\left(\frac{1}{Ls}\right) = -y(s)$

$\Rightarrow$ that circuits made only with L's & C's have odd $y(s)$