EE 610

09/11/12

Note Title 9/7/2012

$$\begin{bmatrix} 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ C_1&0&0&0&0&0\\ 0&0&0&0&0&0\\ C_2&-C_2&0&0&0&0\\ 0&0&0&0&0&-L_6 \end{bmatrix}\frac{dx}{dt} = \begin{bmatrix} -1&0&0&0&0&0\\ 0&-G&0&-1&1&1\\ 0&0&1&0&0&0\\ -g_m&0&0&1&0&0\\ 0&0&0&0&1&0\\ -1&1&0&0&0&0 \end{bmatrix}x + \begin{bmatrix}1\\0\\0\\0\\0\\0\end{bmatrix}u$$

$$y = [0\ 1\ 0\ 0\ 0\ 0]\,x \qquad x^T = [v_1\, v_2,\, i_3\, i_4\, i_5\, i_6]^T_6$$

$$E\frac{dx}{dt} = Ax + Bu,\quad y = Cx \qquad T(s) = C(Es-A)^{-1}B$$

$$Esx = Ax + Bu$$

$$(Es-A)x = Bu$$

$$x = (Es-A)^{-1}Bu$$

$$y = Cx = C(Es-A)^{-1}B\cdot u$$

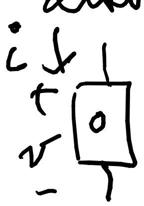
Ex:

$$\begin{bmatrix}1&0\\0&0\end{bmatrix}\frac{dx}{dt} = \begin{bmatrix}0&-1\\-1&0\end{bmatrix}x + \begin{bmatrix}0\\1\end{bmatrix}u,\quad y = [a,\, b]$$

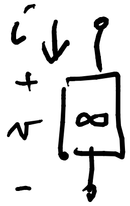
$$(Es-A) = \begin{bmatrix}s&+1\\+1&0\end{bmatrix} \Rightarrow (Es-A)^{-1} = \frac{1}{-1}\begin{bmatrix}0&-1\\-1&s\end{bmatrix} = \begin{bmatrix}0&1\\1&-s\end{bmatrix}$$

$$T(s) = [a,\, b]\begin{bmatrix}0&1\\1&-s\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix} = [b,\ a - sb]\begin{bmatrix}0\\1\end{bmatrix} = a - sb$$

also look at nullators & norators



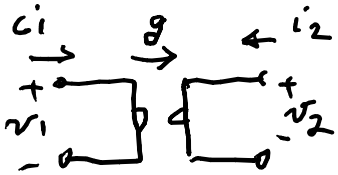
= nullator $\Rightarrow i = 0,\ v = 0$ $\qquad \begin{bmatrix}0\\1\end{bmatrix}[v] = \begin{bmatrix}1\\0\end{bmatrix}[i]$



= norator $\Rightarrow$ i & v arbitrary & independent

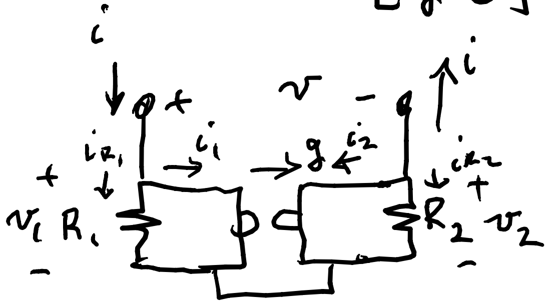
$$[0][v] = [0][i] \Rightarrow \text{norator}$$

Gyrators



$$\begin{bmatrix} 0 & -g \\ g & 0 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

$$Y_{2\times 2} = \begin{bmatrix} 0 & -g \\ g & 0 \end{bmatrix}$$



$$v = v_1 - v_2$$

$$g v_1 = i_2 \Rightarrow g R_1 i_{R_1} = g R_1 (i - i_1)$$

$$-g v_2 = i_1 \Rightarrow -g R_2 i_{R_2} = -g R_2 (-[i + i_2])$$

$G_1 = \frac{1}{R_1}$

$G_2 = \frac{1}{R_2}$

$$\left.\begin{aligned} v_1 &= \frac{i_2}{g} = \frac{-i - i_{R_2}}{g} = -\frac{i}{g} - \frac{v_2}{gR_2} \\ v_2 &= -\frac{i_1}{g} = -\frac{1}{g}(i - i_{R_1}) = -\frac{i}{g} + \frac{v_1}{gR_1} \end{aligned}\right\} \quad v_1 - v_2 = -\frac{v_2}{gR_2} - \frac{v_1}{gR_1}$$

$$v_1 + \frac{v_1}{gR_1} = v_2 - \frac{v_2}{gR_2}$$

if $R_1 = -R_2 \Rightarrow v_1 = v_2 \Rightarrow v = 0 \Leftarrow$ $v_1\left(1 + \frac{1}{gR_1}\right) = v_2\left(1 - \frac{1}{gR_2}\right)$

if $R_1 = +\frac{1}{g} \Rightarrow 0 = v_2\left(1 - \frac{1}{gR_2}\right) = v_2\left(1 + \frac{1}{gR_1}\right) = v_2\left(1 + \frac{1}{g^2}\right)$

if $\cancel{gR_2 \neq 1} \Rightarrow v_2 = 0 \Rightarrow v_1 = 0$

$\Rightarrow \cancel{v_1 = -i/g}$; $v_2 = 0 \Rightarrow 0 = -\frac{i}{g} + \frac{v_1}{gR_1} = -\frac{i}{g} + \frac{i}{g^2 \cancel{R_1}} = -\frac{i}{g}\left(1 + \frac{1}{g\cancel{R_1}}\right)$

if $g\cancel{R_1} \cancel{\neq} -1$ then $i = 0$

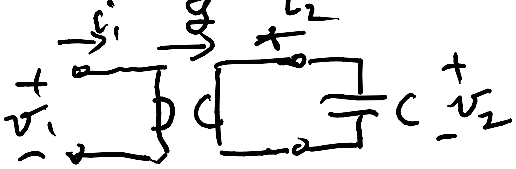
& $v_1 = -i/g = 0$

$\therefore$ if $gR_1 = +1$ & $gR_2 = -1 \Rightarrow$ this is a nullator

as $v = 0$ & $i = 0$

if $gR_1 = -1$ & $gR_2 = 1$ then v_2 is arbitrary $\Rightarrow v_1$ is arbitrary & i is arbitrary

Some equivalences

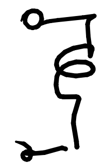


$$i_1 = -g v_2$$

$$i_2 = g v_1 = -sC v_2$$

$$\frac{i_1}{g v_1} = \frac{-g v_2}{-sC v_2} \Rightarrow \frac{i_1}{v_1} = \frac{g^2}{sC}$$

$$v_1 = \frac{sC}{g^2} \cdot i_1$$



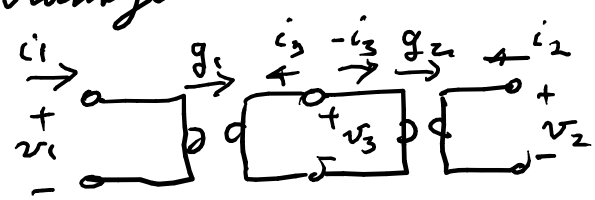
$L = C/g^2$

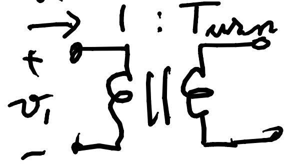
$g \sim 10^{-3}$ ℧

$C \sim \mu Fd = 10^{-6}$

$$L = \frac{10^{-6}}{(10^{-3})^2} = 1 \text{ Hy}$$

Ideal transformer





$v_2 = T_{urn} v_1$

$i_1 = -g_1 v_3$ $\qquad$ $-i_3 = -g_2 v_2$ $\qquad$ $1 \cdot i_1 + T_{urn} i_2 = 0$

$i_3 = g_1 v_1$ $\qquad$ $i_2 = g_2 v_3$ $\qquad$ $v_2 = T_{urn} v_1$

$i_1 = -T_{urn} i_2$

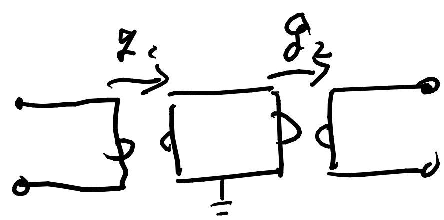
$i_1 = -g_1 \left(\frac{i_2}{g_2}\right)$ & $i_3 = g_1 v_1 = g_2 v_2$

$= -\frac{g_1}{g_2} \cdot i_2$ $\qquad$ $v_2 = \frac{g_1}{g_2} \cdot v_1$

$= -T_{gy} \cdot i_2$ $\qquad$ $= T_{gy} v_1$

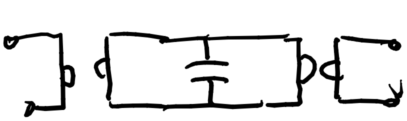
$$\begin{bmatrix} -T_{urn} & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 1 & T_{urn} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

$$T_{gy} = \frac{g_1}{g_2}$$

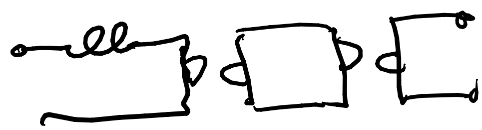


$\equiv$

$1 : g_1/g_2$



$\equiv$



$\equiv$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ C_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ C_2 & -C_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -L_6 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix}$$

$$\overbrace{\begin{bmatrix} 0 & 0 & 0 & 0 & 6 \\ 0 & 0 & 0 & 0 & 0 \\ c_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -c_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -L6 \end{bmatrix}}^{11} \frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 - x_1 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \underbrace{\begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 0 \\ -G & -G & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ -gm & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}}_{t_2 \quad t_{12}^{-1}}^{\underline{1}_6} \begin{bmatrix} x_1 \\ x_2 - x_1 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} u$$

$$y = [1 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0]\, x_{new}$$

continue to get $\begin{bmatrix} \underline{1}_3 & 0_3 \\ 0 & 0_e \end{bmatrix}$