EE610
09/06/12
→ 09/09/12

$$(e Y_{b\times b} E - eJ) = e Y_{b\times b} e^T \cdot v_t = \begin{bmatrix} G_i + g_m + g_\pi + sC_\pi + G_L & -G_L \\ -g_m - G_L & sC_\mu + G_L \end{bmatrix} v_t$$

$$v_t = \left[e Y_{b\times b} e^T\right]^{-1} \times (e Y_{b\times b} E - eJ)$$

$$e Y_{b\times b} E = \begin{bmatrix} G_i + g_m & 0 & g_\pi + sC_\pi & 0 & G_L \\ -g_m & sC_\mu & 0 & 0 & -G_L \end{bmatrix} \begin{bmatrix} v_i \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} G_L + g_m \\ -g_m \end{bmatrix} v_i$$

$$\Rightarrow \begin{bmatrix} G_i + g_m + g_\pi + sC_\pi + G_L & -G_L \\ -g_m - G_L & sC_\mu + G_L \end{bmatrix} v_t = \begin{bmatrix} G_L + g_m \\ -g_m \end{bmatrix} v_i$$

$$s \begin{bmatrix} C_\pi & 0 \\ 0 & C_\mu \end{bmatrix} v_t = \begin{bmatrix} -G_i - g_m - g_\pi - G_L & G_L \\ g_m + G_L & G_L \end{bmatrix} v_t + \begin{bmatrix} G_L + g_m \\ -g_m \end{bmatrix} v_i$$

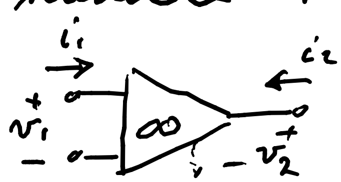
of the form $\frac{dv_t}{dt} = A v_t + B v_i \Rightarrow$ state variable eqs.

Let $v_5 = v_{output}$ $\quad v_5 = v_1 - v_2 = [1 \; -1] \, v_t = C v_t = y$

$$\frac{dx}{dt} = Ax + Bu$$
$$y = Cx$$

More general, to handle op-amps, L's

op-amp ideal

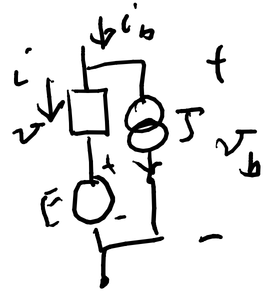


$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

$v_1 \equiv 0$ (virtual "gnd")

$i_1 \equiv 0$ (∞ input Z)

$A(s)\cdot v = B(s)\, i$ components.



$v = v_b - E$

$i = i_b - J$

$v_b = \mathcal{C}^T v_t \,,\quad i_b = \mathcal{D}^T i_\ell$

$A(s)\, v_b - AE = B(s)\, i_b - BJ$

$A(s)\,\mathcal{C}^T v_t - B(s)\,\mathcal{D}^T i_\ell = AE - BJ$

$$\left[\, A(s)\,\mathcal{C}^T \;\vdots\; -B(s)\,\mathcal{D}^T \,\right]\begin{bmatrix} v_t \\ i_\ell \end{bmatrix} = AE - BJ$$

b rows → ; t columns ; l column ; b×b matrix

$x = \begin{bmatrix} v_t \\ i_\ell \end{bmatrix}$ is a b-vector — full branch entries

u = sources

⇓

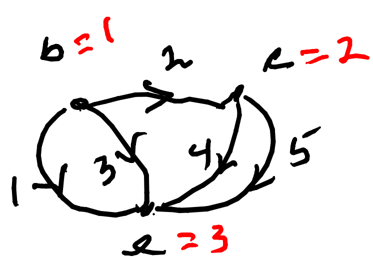
$$s E_s x = A_s x + B_s u$$
$$y = C_s x$$

} semi-state eqs.

here E_s is generally singular; if nonsingular, multiply by E_s^{-1} to get state variable eqs.

— — to get the # of possible trees, form incidence matrix A_a

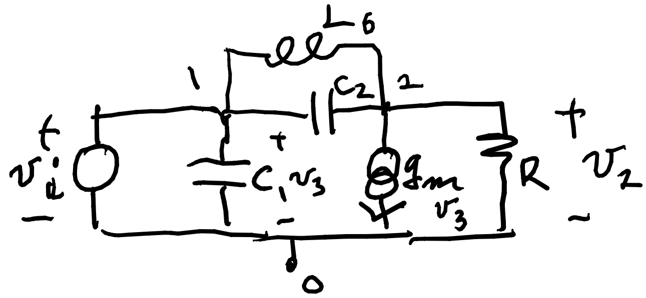
sum currents at a node to be 0

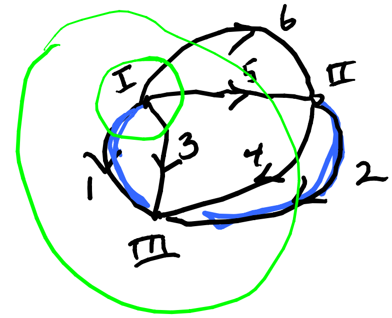


$$A_a = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & -1 & 0 & +1 & +1 \\ -1 & 0 & -1 & -1 & -1 \end{bmatrix} \Rightarrow A = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & -1 & 0 & 1 & 1 \end{bmatrix}$$

$$\det AA^T = \det \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & -1 & 0 & 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 1 & -1 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{bmatrix} = \det \begin{bmatrix} 3 & -1 \\ -1 & 3 \end{bmatrix} = 9 - 1 = 8$$

Ex. for semistate eqs.





| = tree

$$u = v_i, \quad y = v_2, \quad x = \begin{bmatrix} v_t \\ i_l \end{bmatrix} \begin{matrix} \}2 \\ \}4 \end{matrix} = \begin{bmatrix} v_1 \\ v_2 \\ i_3 \\ i_4 \\ i_5 \\ i_6 \end{bmatrix}_b \begin{matrix} \}\text{tree} \\ \\ \}\text{link} \end{matrix}$$

$$J \equiv 0, \quad E = \begin{bmatrix} v_i \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$A(s)v = B(s)i$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & G & 0 & 0 & 0 & 0 \\ 0 & 0 & sC_1 & 0 & 0 & 0 \\ 0 & 0 & g_m & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & sC_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & sL_6 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ i_6 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \left[\begin{array}{cc|cccc} 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & -1 & -1 \end{array}\right] i_b = \underline{0} = C i_b, \quad \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \left[\begin{array}{cc|cccc} -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 & 0 \\ -1 & 1 & 0 & 0 & 1 & 0 \\ -1 & 1 & 0 & 0 & 0 & 1 \end{array}\right] v_b = \underline{0} = J v_b$$

$$AC^T = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & G & 0 & 0 & 0 & 0 \\ 0 & 0 & sC_1 & 0 & 0 & 0 \\ 0 & 0 & g_m & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & sC_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \\ 1 & -1 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & G \\ sC_1 & 0 \\ g_m & 0 \\ sC_2 & -sC_2 \\ 1 & -1 \end{bmatrix}$$

$$-BJ^T = -\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & sL_6 \end{bmatrix} \begin{bmatrix} -1 & 0 & -1 & -1 \\ 0 & -1 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = -\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & sL_6 \end{bmatrix}$$

$$\left[\begin{array}{cc|c|cccc} 1 & 0 & - & 0 & 0 & 0 & 0 \\ 0 & G & & 0 & -1 & 1 & 1 \\ sC_1 & 0 & & 1 & 0 & 0 & 0 \\ g_m & 0 & & 0 & 1 & 0 & 0 \\ sC_2 & -sC_2 & & 0 & 0 & 1 & 0 \\ 1 & -1 & & 0 & 0 & 0 & sL_G \end{array}\right] = \left[AE^T \,\vdots\, -BJ^T\right]$$

$$\Rightarrow \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ C_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ C_2 & -C_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -L_G \end{bmatrix} \frac{dx}{dt} = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -G & 0 & -1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ -g_m & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ -1 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} x + \overset{\overset{AE}{\downarrow}}{\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}} v_i^{=u}$$

$$y = v_2 = [0\ 1\ 0\ 0\ 0\ 0]\, x$$

names of these equations:
- descriptor
- singular
- differential-algebraic
- DAE
- semi-state