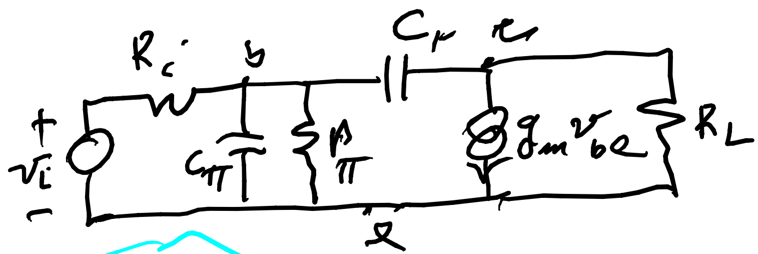
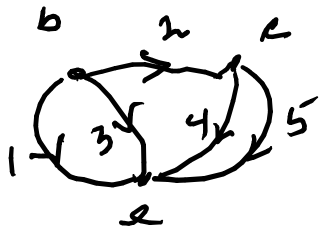
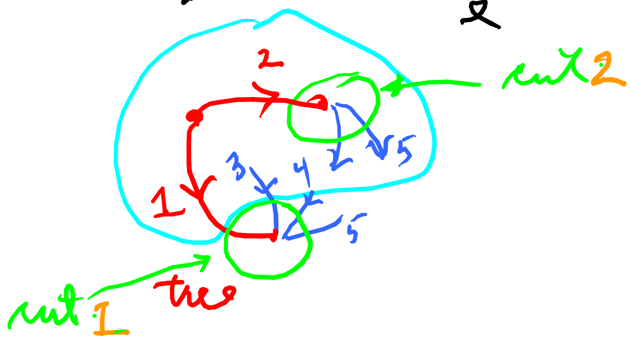
Note Title 9/4/2012

EE610
09/04/12







note

this has the same graph
with v_1 & v_2 can be independent
i_3, i_4, i_5 " " "

} the tree branch voltages can be "variables" & same for cotree branch currents

write i_b = branch currents = $\begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \end{bmatrix} = i_b$, $v_b = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{bmatrix} = \begin{bmatrix} v_t \\ \hline v_\ell \end{bmatrix}$

v_b = branch voltages

$$= \begin{bmatrix} i_t \\ i_\ell \end{bmatrix}$$

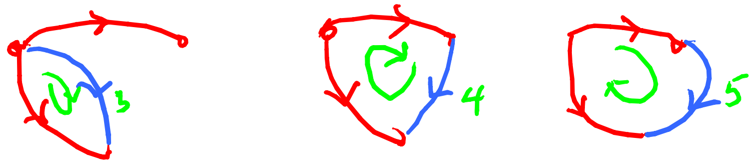
KCL:

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \left[\begin{array}{cc|ccc} 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & -1 & -1 \end{array}\right] \begin{bmatrix} i_1 \\ i_2 \\ \hline i_3 \\ i_4 \\ i_5 \end{bmatrix}$$ = cut set equations

$$0_t = C i_b$$

$$= \left[\begin{array}{c|c} 1_2 & -K \end{array}\right] \begin{bmatrix} i_t \\ \hline i_\ell \end{bmatrix} \Rightarrow i_t = K i_\ell \Rightarrow i_b = \begin{bmatrix} i_t \\ i_\ell \end{bmatrix} = \begin{bmatrix} -K \\ \hline 1_\ell \end{bmatrix} i_\ell$$

KVL: use loops



$$\begin{bmatrix}0\\0\\0\end{bmatrix} = \left[\begin{array}{cc|ccc}-1 & 0 & 1 & 0 & 0\\ -1 & 1 & 0 & 1 & 0\\ -1 & 1 & 0 & 0 & 1\end{array}\right]\begin{bmatrix}v_1\\v_2\\v_3\\v_4\\v_5\end{bmatrix} \Rightarrow 0_\ell = \mathcal{T} v_b ,$$

$\mathcal{T}$ = tie set matrix

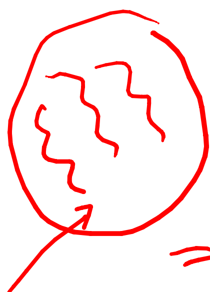
$\mathcal{C}$ = cut set matrix

$$= \left[K^T \,\middle|\, 1_3 \right]\begin{bmatrix}v_t\\v_\ell\end{bmatrix} \Rightarrow v_\ell = -K^T v_t \Rightarrow \underbrace{\begin{bmatrix}v_t\\v_\ell\end{bmatrix}}_{v_b} = \begin{bmatrix}1_t\\ \hline -K^T\end{bmatrix} v_t = \mathcal{C}^T v_t$$

$v_b = \mathcal{C}^T v_t$, $\quad 0_t = \mathcal{C} i_b$

$i_b = \mathcal{T}^T i_\ell$; $\quad 0_\ell = \mathcal{T} v_b$

note if know K, know both $\mathcal{C}$ & $\mathcal{T}$ so KVL ⇔ KCL



circuit

$\Rightarrow$ Power in from outside $= 0 = \sum_{i=1}^{b} v_b i_b = v_b^T i_b = v_b^T \mathcal{T}^T i_\ell = v_t^T \mathcal{C} \mathcal{T}^T i_\ell$

$$\Rightarrow v_b^T \mathcal{T} = 0_\ell$$

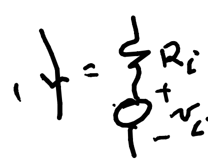
$$\Rightarrow \mathcal{C} \mathcal{T}^T = 0_{t,\ell}$$

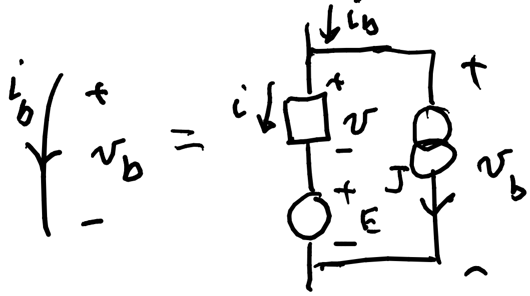
$$\left[1_t \,\middle|\, -K\right]\begin{bmatrix}Q\\ \hline 1_\ell\end{bmatrix} \Rightarrow Q = K \Rightarrow \mathcal{T}^T = \begin{bmatrix}Q\\ 1_\ell\end{bmatrix}$$

$$\mathcal{T} = \left[Q^T \;\; 1_\ell\right]$$
$$= \left[-K^T \,\middle|\, 1_\ell\right]$$

for laws of components use

$$Y_{b\times b} = \begin{bmatrix} G_i & 0 & 0 & 0 & 0\\ 0 & sC_\mu & 0 & 0 & 0\\ 0 & 0 & g_\pi + sC_\pi & 0 & 0\\ g_m & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & G_L \end{bmatrix}$$





$v_b = v + E \qquad i = Y_{b\times b}\, v$

$i_b = i + J$

$$i = i_b - J = Y_{b\times b} \cdot v = Y_{b\times b}(v_b - E) \Rightarrow$$

$$i_b = Y_{b\times b} \times v_b + (J - Y_{b\times b} E)$$

$$i_b = e^T i_\ell, \quad 0 = e\, i_b = 0 = e\, Y_{b\times b}\, e^T \cdot v_t + e(J - Y_{b\times b} E)$$

$$\Rightarrow (e\, Y_{b\times b} E - e J) = e\, Y_{b\times b}\, e^T \cdot v_t$$

t equations for t unknowns (v_t)

$$\Rightarrow v_t = \left[e\, Y_{b\times b}\, e^T\right]^{-1} \times \underbrace{(e\, Y_{b\times b} E - e J)}_{\text{Norton's equivalent current sources feeding tree branches}}$$

$$\left[\begin{array}{cc|ccc} 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & -1 & -1 \end{array}\right] \begin{bmatrix} G_i & 0 & 0 & 0 & 0 \\ 0 & sC_\mu & 0 & 0 & 0 \\ 0 & 0 & g_\pi + sC_\pi & 0 & 0 \\ g_m & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & G_L \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 1 & -1 \\ 1 & -1 \end{bmatrix} = e\, Y_{b\times b}\, e^T$$

$$= \begin{bmatrix} G_i + g_m & 0 & g_\pi + sC_\pi & 0 & G_L \\ -g_m & sC_\mu & 0 & 0 & -G_L \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 1 & -1 \\ 1 & -1 \end{bmatrix} =$$

$$= \begin{bmatrix} G_i + g_m + g_\pi + sC_\pi + G_L & -G_L \\ -g_m - G_L & sC_\mu + G_L \end{bmatrix}$$