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610 Fall 2012 – Homework 6

1. Synthesize by the two Cauer and the two Foster forms the lossless admittances

$$y_1(s) = \frac{s(s^2+3)}{(s^2+1)(s^2+5)}$$

$$y_2(s) = \frac{(s^2+1)(s^2+5)}{s(s^2+3)}$$

2. Replace each L by an R in the above syntheses and give the resulting RC admittances. Compare degrees of different y(s), including LC versus RC.

3. Set up the companion matrix form of state-variable equations for the admittance

$$y(s) = \frac{3s^2+2s+1}{s^2+2}$$

and synthesize the resulting 3-port admittance matrix, loaded in two unit capacitors, and check by analyzing the circuit that y(s) results.

4. From Homework #5, problem 5, the following formulas hold for the lossless positive-real tanh and cotanh=coth.

$$\tanh(s) = 2s[\sum_{n=0}^{\infty} \frac{1}{(s^2+(n+\frac{1}{2})^2\pi^2)}]$$

$$\coth(s) = \frac{1}{s} + 2s[\sum_{n=1}^{\infty} \frac{1}{(s^2+(n\pi)^2)}]$$

Using these give second Foster and second Cauer type syntheses of y(s)=coth(sl) [which represents a normalized shorted transmission line of length l].

5. (conceptually challenging). Show that the following is positive-real and discuss the position and nature of its singularities (note that a limit of poles is an essential singularity and all real numbers are limits of rational numbers).

$$y(s) = \sum_{k=1}^{\infty} \sum_{m=1}^{\infty} \coth(s\frac{k}{m})$$