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610 Fall 2012 – Homework 5

1. a) Prove that a positive-real function of a positive-real function is positive-real and also that a PR function of a PR function is PR.
   b) Evaluate y1(y2(s)) for the PR functions

$$y1(s) = \frac{3s}{s^2+2} + 5$$

$$y2(s) = \frac{4s}{s^2+1} + 3$$

   c) Synthesize the admittances y1 & y2 of b) and from those synthesize y1(y2(s))

2. For the admittance

$$y(s) = \frac{s(s^2+as+b)}{(s^2+4)(s+c)}$$

a) Give conditions on the constants a, b, c such that y(.) is PR. Include the separate case of c=0.
b) Synthesize the PR y(s).
c) In the case c=0 discuss what will change if one were to synthesize the non-PR y(s).

3. a) Synthesize, using the Richards' function and gyrator-C, 2-ports in cascade the lossless admittance $y(s) = \frac{2s}{s^2+2}$. Use k=1 where possible and discuss the effect of using possibly different k.
   b) Compare with the Cauer and Foster forms.
   c) Repeat a) on the PR function $y(s) = \frac{2s}{s^2+2} + 4$

4. Prove or disprove that for any positive-real function with poles on the jω axis there will be jω axis zeroes such that poles and zeroes will alternate.

5. The following formulas hold for tanh and cotanh=coth.

$$\tanh(s) = 2s\left[\sum_{n=0}^{\infty} \frac{1}{(s^2+(n+\frac{1}{2})^2\pi^2)}\right]$$

$$\coth(s) = \frac{1}{s} + 2s\left[\sum_{n=1}^{\infty} \frac{1}{(s^2+(n\pi)^2)}\right]$$

Give their poles and zeroes and show that these are lossless positive-real functions, though not PR.