12/17/12
$r_{1,1}$

EMEE 610 Fall 2012 Final

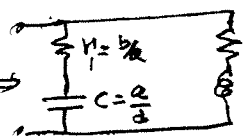
#1, a) $2\,Ev\, y(s) \doteq y(s) + y(-s) = \dfrac{as+b}{cs+d} + \dfrac{-as+b}{-cs+d} = \dfrac{(as+b)(-cs+d)+(-as+b)(cs+d)}{(cs+d)(-cs+d)}$

$= \dfrac{(-acs^2+bd+ads-bcs)+(-acs^2+bd-ads+bcs)}{-c^2s^2+d^2}$

$= 2(-acs^2+bd)/(d^2-c^2s^2) \Rightarrow Ev\, y(s)=0 \;@\; -acs^2+bd=0$

$\Rightarrow$ zeros of $Ev\, y(s)$ @ $s = \pm\sqrt{\dfrac{bd}{ac}}$ (which are real as $a, b, c, d > 0$)

b) $y(s) = \dfrac{as}{bs+d} + \dfrac{b}{cs+d} = \dfrac{1}{\dfrac{b}{a} + \dfrac{1}{\frac{a}{d}s}} + \dfrac{1}{\frac{c}{b}s + d/b}$



as all of r_1, r_2, C, L are >0 this is a passive circuit which requires $y(s)$ to be PR

$\therefore$ $y(s)$ is PR for all positive a, b, c, d.

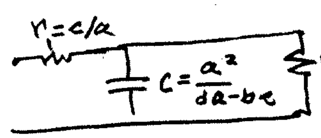
other means of checking: poles are in $\sigma<0 \Rightarrow$ analytic in $\sigma>0$ and $G(j\omega) = Ev\, y(j\omega) = (ac\omega^2+bd)/(d^2+c^2\omega^2) > 0 \;\forall\omega$

c) For $y(s)$ to RC, PR, poles and zeros on $-\sigma$ axis (true for all $a,b,c,d>0$) and alternate with highest a pole & lowest a zero. Write $y(s) = \dfrac{a}{c}\left(\dfrac{s+b/a}{s+d/c}\right)$, so need $d/c \geq b/a$ [the RC condition].

d) 1st Cauer: remove poles of $y(s)$@∞ and constants of z@∞ as no pole at ∞, start with $z = (cs+d)/(as+b)$

$$\begin{array}{l} as+b \,\big)\, cs+d \;\Big|\; \frac{c}{a} \\ \quad cs + b\cdot c/a \\ \quad\quad \frac{da-bc}{a} \,\big)\, as+b \;\Big|\; \frac{a^2}{da-bc}s \\ \quad\quad\quad as \\ \quad\quad\quad\quad b \,\big)\, \frac{da-bc}{a} \;\Big|\; \frac{ba}{da-bc} \end{array}$$

$\Rightarrow y(s) = \dfrac{1}{\dfrac{c}{a} + \dfrac{1}{\dfrac{a^2}{da-bc}s + \dfrac{1}{\frac{ba}{da-bc}}}}$

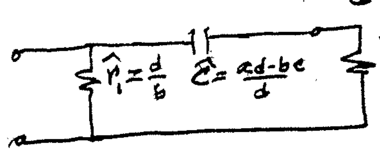


; $C>0$ & $r_2 \geq 0$ by RC condition

2nd Cauer: remove constants of y@0 and poles of z@0

$$\begin{array}{l} d+cs \,\big)\, b+as \;\Big|\; b/d \\ \quad b+\frac{cb}{d}s \\ \quad\quad \frac{ad-bc}{d}s \,\big)\, d+cs \;\Big|\; \frac{d}{ad-bc}\cdot\frac{1}{s} \\ \quad\quad\quad d \\ \quad\quad\quad\quad cs \,\big)\, \frac{ad-bc}{d}s \;\Big|\; \frac{ad-bc}{d^2} \end{array}$$

$\Rightarrow y(s) = \dfrac{b}{d} + \dfrac{1}{\dfrac{d}{ad-bc}\cdot\dfrac{1}{s} + \dfrac{1}{\frac{ad-bc}{d^2}}}$



& $C \geq 0$, $r_2 > 0$

#2. The 2-port $Y(s) = \begin{bmatrix} Cs+G & -Cs-G-g \\ -Cs-G+g & Cs+G \end{bmatrix}$ & $y = y_{11} - \dfrac{y_{12}y_{21}}{y_{22}+y_L}$
$G = 1/R$

$$= \frac{\Delta y + y_{11}y_L}{y_{22}+y_L} = \frac{g^2 + (Cs+G)y_L}{(Cs+g)+y_L}$$

a) $\Rightarrow \; y\,y_L + y(Cs+G) = g^2 + (Cs+G)y_L$

$$y_L = \frac{g^2 - (Cs+\frac{1}{R})y}{y - (Cs+\frac{1}{R})}$$

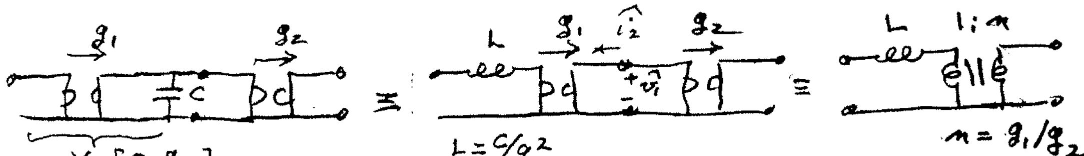
b) as $Cs + \frac{1}{R} = C(s+a)$ has the same a for all C's we can set $s+a = p \Rightarrow s = p-a$ and synthesize $\hat{y}(p) = y(p-a)$ (which is assumed PR) and then replace each Cp in the circuit by $Cp = C(s+a) \Rightarrow$ 

12/17/12

ENEE 610 Fall 2012 Final

#3

#3, a)



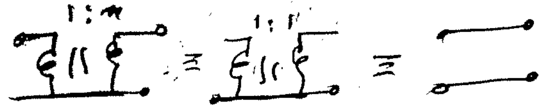
$\underbrace{\quad\quad}_{Y = \begin{bmatrix} 0 & g_1 \\ -g_1 & sC \end{bmatrix}}$ $\quad L = C/g_1^2 \quad$ $n = g_1/g_2$

$\equiv Z = \begin{bmatrix} sC/g_1^2 & 1/g_1 \\ -1/g_1 & 0 \end{bmatrix}$

$i_1 = g_1 \hat{v}_1, \; i_2 = -g_2 \hat{v}_1$

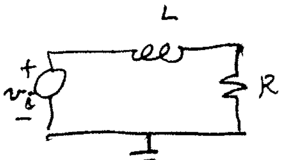
$i_1 = -g_1 i_2 / g_2$

choose $g_1 = g_2$ so

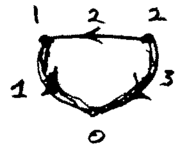


$\Rightarrow L = C/g_1^2$
$g_2 = g_1$

b)



$\equiv$



$v_t = \begin{bmatrix} v_1 \\ v_3 \end{bmatrix}, \; v_b^T = [v_1, v_2, v_3]$

$i_\ell = [i_2], \quad i_b^T = [i_1, i_2, i_3]$

$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \mathcal{C} i_b = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 1 \end{bmatrix} i_b$

$[0] = \mathcal{J} v_b = [1 \;\; 1 \;\; -1] v_b$

$\Rightarrow v_b = \mathcal{C}^T v_t = \begin{bmatrix} 1 & 0 \\ -1 & 1 \\ 0 & 1 \end{bmatrix} v_t \qquad i_b = \mathcal{J}^T i_\ell = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} i_\ell$

$v_b = \begin{bmatrix} v_i \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ v_L \\ v_R \end{bmatrix}; \; i_b = \begin{bmatrix} i_{v_i} \\ i_L \\ i_R \end{bmatrix}$

$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}_b = \begin{bmatrix} v_i = u \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ v_L = sL i_L = sL i_2 \\ v_R = R i_R = R i_3 \end{bmatrix} = \begin{bmatrix} u \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & sL & 0 \\ 0 & 0 & R \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ -1 & 1 \\ 0 & 1 \end{bmatrix} v_t = \begin{bmatrix} u \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & sL & 0 \\ 0 & 0 & R \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} i_L = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} u + \begin{bmatrix} 0 \\ sL \\ R \end{bmatrix} i_L$

$\Rightarrow \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = -\left[\begin{array}{cc|c} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 1 & 0 \end{array}\right] \begin{bmatrix} v_t \\ i_\ell \end{bmatrix} + \left[\begin{array}{cc|c} 0 & 0 & 0 \\ 0 & 0 & sL \\ 0 & 0 & -R \end{array}\right] \begin{bmatrix} v_t \\ i_\ell \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} u$

$\Rightarrow \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -L \\ 0 & 0 & 0 \end{bmatrix} \frac{dx}{dt} = \begin{bmatrix} -1 & 0 & 0 \\ 1 & -1 & 0 \\ 0 & -1 & -R \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} u \quad , \; y = v_0 = v_3 = [0 \;\; 1 \;\; 0] \begin{bmatrix} v_1 \\ v_3 \\ i_2 \end{bmatrix}$

$y = [0 \;\; 1 \;\; 0] x$

Here $x_1 = u, \quad x_2 = -R x_3, \quad -Ls x_3 = x_1 - x_2 = u + R x_3 \Rightarrow x_3 = \frac{-u}{R + Ls}$

$y = v_0 = x_2 = \frac{R}{R + Ls} \cdot u$

$\Rightarrow \frac{v_0}{v_i} = \frac{R}{R + Ls}$ (checks by voltage divider equation applied to circuit)