EE610
10/25/11

Note Title 10/22/2011

$$S_x^T = \frac{\partial T/\partial x}{T/x} \quad ; \quad T(x) = |T(x)| e^{j\angle T}$$

$$= \frac{\partial |T|/\partial x \cdot e^{j\angle T}}{|T| e^{j\angle}/x} + \frac{|T|\, de^{j\angle T}/dx}{|T| e^{j\angle T}/x}$$

$$= S_x^{|T|} + \frac{de^{j\angle x}/dx}{e^{j\angle}/x} = S_x^{|T|} + j\frac{e^{j\angle T}\, d\angle T/dx}{e^{j\angle T}/x}$$

$$= S_x^{|T|} + j\frac{d\angle T/dx}{1/x} = \mathrm{Re}\, S_x^T + j\, \mathrm{Im}\, S_x^T$$

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e^{At}, $A = n \times n$ $\Rightarrow$ $E\dfrac{dx}{dt} = A(x,t) + Bu$

$y = Cx$

often can reduce to state variable equations

$$\frac{d\hat{x}}{dt} = \hat{A}\hat{x} + \hat{B}u \qquad \Rightarrow E \text{ nonsingular}$$

$$y = \hat{C}\hat{x} + \hat{D}u \qquad \hat{A} \text{ square}$$

if $u = 0$ $\quad \dfrac{d\hat{x}}{dt} = \hat{A}\hat{x}$, $\quad \hat{x}(0)$ given

$$\hat{x} = e^{\hat{A}t} X \quad ; \quad \hat{x}(0) = e^{A \cdot 0} X = X$$

$$e^{At} \triangleq \sum_{i=0}^{\infty} \frac{(At)^i}{i!} \quad ; \quad \frac{de^{At}}{dt} = \sum_{i=0}^{\infty} \frac{A^i t^{i-1}}{i!} \times i = A \sum_{i=1}^{\infty} \frac{A^{i-1} t^{i-1}}{(i-1)!}$$

$$= A \sum_{i=0}^{\infty} \frac{(At)^i}{i!} = Ae^{At}$$

If $A = \mathrm{diag}[a_k] = \begin{bmatrix} a_1 & & & \\ & a_2 & & \\ & & \ddots & \\ & & & a_n \end{bmatrix}$

Then $e^{At} = \begin{bmatrix} e^{a_1 t} & & & \\ & e^{a_2 t} & & \\ & & \ddots & \\ & & & e^{a_n t} \end{bmatrix}$

$$\frac{dx}{dt} = Ax + Bu$$
$$y = Cx + Du$$

then we let

$$\tilde{A} = TAT^{-1}, \quad T = \text{const.}$$
$$\bar{x} = Tx \Rightarrow x = T^{-1}\bar{x}$$

$$\frac{d\bar{x}}{dt} = T\frac{dx}{dt} = TAT^{-1}\bar{x} + TBu$$
$$y = CT^{-1}\bar{x} + Du$$

a change on the state but preserves the input output map

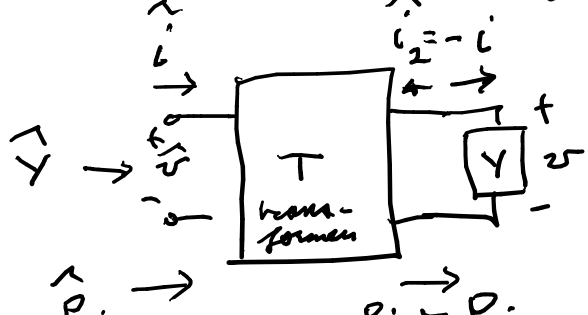
This is a similarity transformation on the state (differs from the energy preserving congruency transformation TAT^T)

If the $T^T = T^{-1}$ then these agree; ie if T is orthogonal. Similarity transformations allow isolation of eigenvalues; if all are different then A can be diagonalized. If not distinct go to Jordan canonical form.

$$P = v^T i, \qquad i = Yv;$$
$$= v^T Y v \qquad \text{if } v = \hat{T}\hat{v}$$
$$= \hat{v}^T \hat{T}^T Y T \hat{v} \Rightarrow \hat{Y} = \hat{T}^T Y \hat{T} \text{ has power conserved}$$

$$i = Yv$$
$$= Y\hat{T}\hat{v} \Rightarrow \hat{T}^T i = \hat{T}^T Y \hat{T}\hat{v} = \hat{i}$$

$$\hat{i} = \hat{Y}\hat{v}$$ here $\hat{T}$ is a transformer turns ratio matrix



$$P_{in\,load} = \hat{P}_{in}$$
$$= -P_{ports\,2\,of\,T}$$

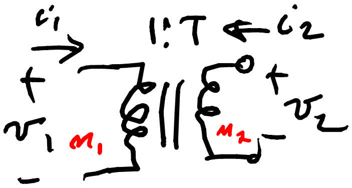
$$v = \hat{T}\hat{v}$$
$$-\hat{i} = \hat{T}^T i \Rightarrow \hat{i} + \hat{T}^T i = 0$$
$$v = \hat{T}\hat{v}$$

lossless multiport

$$v^T i + \hat{v}^T \hat{i} = \hat{v}^T \hat{T}^T i_2 + \hat{v}^T \hat{i}$$
$$= \hat{v}^T \hat{T}^T (-i) + \hat{v}^T \hat{i}$$

$$\hat{v}^T \hat{i} + v^T \hat{i}_2 = \hat{v}^T \hat{i} + v^T(-i) =$$

$$\hat{v}^T \hat{i} + v^T(-i) = \hat{v}^T(+\hat{T}^T i) + \hat{v}^T \hat{T}^T(-i) = 0$$



$$v_2 = T v_1$$
$$i_1 + T^T i_2 = 0$$

$$v_1 i_1 + v_2 i_2$$
$$= v_1 i_1 + v_1 T i_2$$
$$= v_1 i_1 + v_1 T(-T^{-1} i_1)$$
$$= v_1(1-1) i_1 \equiv 0$$

if $1/T$ exists then P_{in}

$$v_2^T i_2 = v_1^T T^T i_2$$
$$= v_1^T(-i_1) \Rightarrow \underbrace{v_2^T i_2 + v_1^T i_1}_{P_{in}} = [v_1^T, v_2^T]\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = v^T i \equiv 0$$

Maximally flat approximation

(low pass, max flat at $\omega = 0$)

$s = j\omega$

$$\frac{V_2}{V_1}(s)$$

$\therefore$ means as many derivatives of $\left|\frac{V_2}{V_1}(j\omega)\right|$ as possible are zero. For low pass

$$T(s) = \frac{V_2}{V_1}(s) = \frac{1}{s^n + a_{n-1}s^{n-1} + \dots + a_0}$$

$$|T(j\omega)|^2 = \frac{1}{(-1)^n \omega^{2n} + b_{2n-2}\,\omega^{2n-2} + \dots\ b_0} = \frac{1}{P(\omega^2)}$$

$$\frac{d|T(j\omega)|}{d\omega} = \frac{d\,\frac{1}{P(\omega^2)}}{d\omega} = -\frac{1}{P^2(\omega^2)} \cdot \frac{dP(\omega^2)}{d\omega}$$

$$\frac{dP(\omega^2)}{d\omega} = (-1)^n \cdot 2n\,\omega^{2n-1} + b_{2n-2}(2n-2)\,\omega^{2n-3} + \dots\ b_0 = 0$$

but the $P(\omega^2)$ is a Taylor series, so its coefficients are coefficients of derivatives

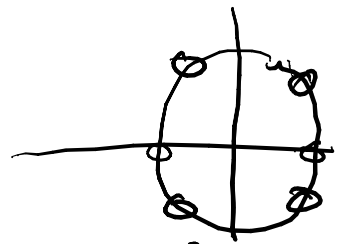
$$P(\omega^2) = b_0 + \underbrace{b_2\,\omega^2 + \dots}_{\text{set these terms to zero}}\ (-1)^n \omega^{2n}$$

$$P(\omega^2) = b_0 + (-1)^n \omega^{2n} = \frac{1}{|T(j\omega)|^2} = \frac{1}{T(s)T(-s)}\Bigg|_{s=j\omega}$$

$$T(s)T(-s) = \frac{1}{b_0 + (-1)^n \left(\frac{s}{j}\right)^{2n}}$$ can normalize $b_0 = 1$ also

$$T(s)T(-s) = \frac{1}{1 + (-1)^n s^{2n}} \Rightarrow$$ poles at $s^{2n} = (-1)^{n+1}$

gives $T(s)$, poles on unit circle & equally spaced



stable choose for $T(s)$

these are for $T(-s)$