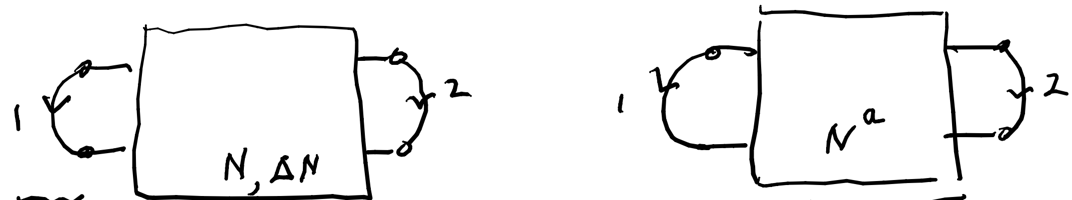
EE610
10/20/11
corrected S_x^T

Sensitivity $\quad S_x^T = \dfrac{\partial T/\partial x}{(T/x)}$



fix

N & N^a have the same graph

fixed

for the graph
from $P_{in}=0 \Rightarrow v^T i = 0 \Rightarrow C\,\vartheta^T = 0_{t\times\ell} \Rightarrow v^T i^a - v^{aT} i = 0$

Δ = change $\quad \Delta(v^T i^a - v^{aT} i) = \Delta v^T \cdot i^a - v^{aT} \Delta i = 0$

keeps N^a fixed

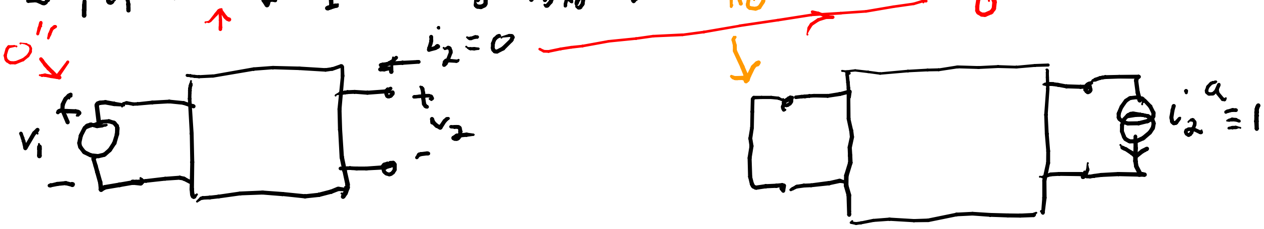
assume that for branches of # > 2 that there is a Y matrix

$$[\Delta v_1, \Delta v_2, \Delta v_b]^T \cdot \begin{bmatrix} i_1^a \\ i_2^a \\ i_b^a \end{bmatrix} - [v_1^a, v_2^a, v_b^a]^T \begin{bmatrix} \Delta i_1 \\ \Delta i_2 \\ \Delta i_b \end{bmatrix} = 0$$

$i_b = Y_{b\times b} v_b, \quad \Delta i_b = \Delta Y_{b\times b} \cdot v_b + Y_{b\times b} \cdot \Delta v_b$

$\Delta v_1 = 0$ as fix branch one as a voltage source feeding N

$$\underset{0}{\Delta v_1 i_1^a} + \Delta v_2 \cdot i_2^a + \Delta v_b^T \cdot Y_{b\times b}^a v_b^a - \underset{0}{v_1^a \Delta i_1} - \underset{0}{v_2^a \Delta i_2} - v_b^{aT} \Delta i_b = 0$$



$$\Delta v_2 \cdot 1 + \Delta v_b^T \cdot Y_{b\times b}^a v_b^a - v_b^{aT} [\Delta Y_{b\times b} v_b + Y_{b\times b} \Delta v_b] = 0$$

$$\Delta v_2 + \Delta v_b^T Y_{b\times b}^a v_b^a - v_b^{aT} \Delta Y_{b\times b} v_b - \Delta v_b^T Y_{b\times b}^T v_b^a = 0$$

set $\Delta v_b^T Y_{b\times b}^a v_b^a - \Delta v_b^T Y_{b\times b}^T v_b^a = \Delta v_b^T \left[Y_{b\times b}^a - Y_{b\times b}^T \right] v_b^a$
$= 0$

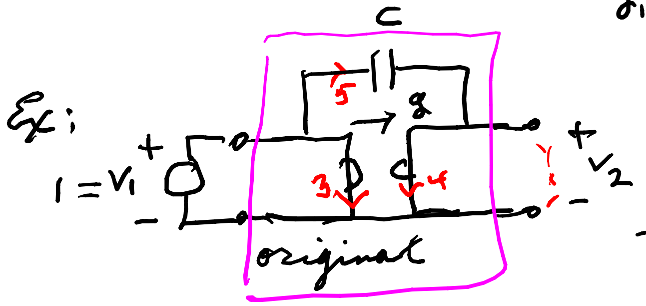
$\Rightarrow$ set $Y_{b\times b}^a = Y_{b\times b}^T$ this defines the adjoint circuit

$\Delta v_2 = + v_b^{aT} \Delta Y_{b\times b} v_b$ if only a g_m changes $g_m = g_{12}$

$\Rightarrow \Delta v_2/1 = + v_{b_1}^a \cdot \Delta g_{12} \cdot v_2$

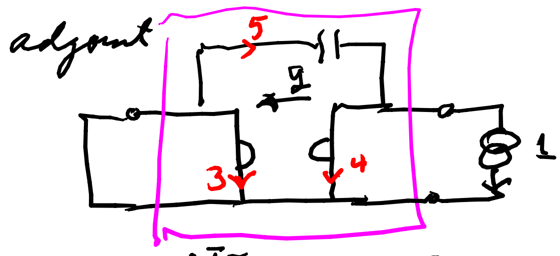
$\Rightarrow \dfrac{\Delta v_2/1 = v_1}{\Delta g_{12}} = + v_{b_1}^a \cdot v_2 = \dfrac{\partial v_2/v_1}{\partial g_{12}}$ in the limit

Ex:



$T(s) = \dfrac{V_2}{V_1}$

$$Y_{b\times b} = \begin{bmatrix} 0 & g & 0 \\ -g & 0 & 0 \\ 0 & 0 & sC \end{bmatrix} \begin{matrix} \leftarrow 3 \\ \leftarrow 4 \\ \leftarrow 5 \end{matrix}$$

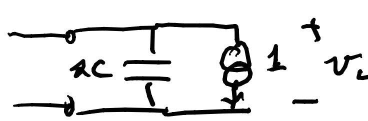


$$Y_{b\times b}^a = Y_{b\times b}^T = \begin{bmatrix} 0 & -g & 0 \\ g & 0 & 0 \\ 0 & 0 & sC \end{bmatrix}$$

$$\Delta v_2 = -v_b^{aT} \begin{bmatrix} 0 & \Delta g & 0 \\ -\Delta g & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} v_b \qquad v_b = \begin{bmatrix} v_3 \\ v_4 \\ v_5 \end{bmatrix}, \quad v_b^a = \begin{bmatrix} v_3^a \\ v_4^a \\ v_5^a \end{bmatrix}$$

$$\frac{\Delta v_2}{\Delta g} = -\begin{bmatrix} v_3^a & v_4^a \end{bmatrix} \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} v_3 \\ v_4 \end{bmatrix} = + \underbrace{v_4^a v_3}_{=1} - \underbrace{v_3^a v_4}_{=0}$$

$v_4^a v_3 = 1$ as $v_3 = v_1$; $v_3^a v_4 = 0$ by $v_1^a = 0$

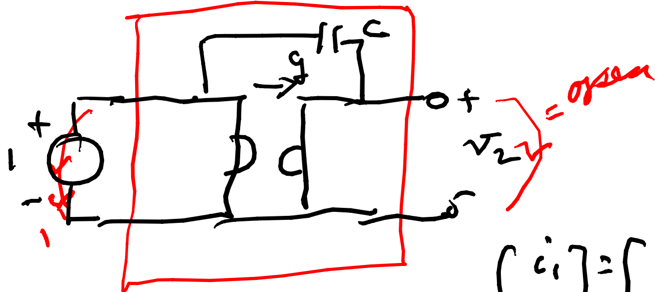


$\Rightarrow -\frac{g}{c} \times \frac{1}{} \Rightarrow v_4^a = -\frac{1}{sC}$

$\dfrac{\partial v_2}{\partial g} = + \dfrac{1}{sC}$; $\quad S_g^{v_2/v_1} = \dfrac{\partial (v_2/v_1)/\partial g}{(v_2/v_1/g)} = + \dfrac{1}{sC} \times \dfrac{g}{(1 + g/sC)} = \dfrac{1}{1 + sC/g}$

see v_2/v_1 below

as a check analyze for $T(s) = v_2/v_1$



$$Y_{2\text{-port}} = \begin{bmatrix} sC & -sC+g \\ -sC-g & sC \end{bmatrix}$$

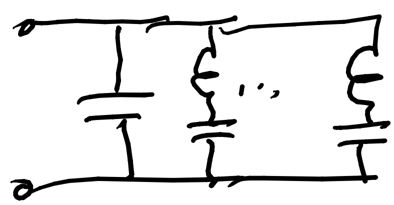
$$0 = \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} & Y & \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \Rightarrow 0 = y_{21} v_1 + y_{22} v_2$$

$$\frac{v_2}{v_1} = -\frac{y_{21}}{y_{22}} = \frac{-(-sC-g)}{sC} = 1 + \frac{g}{sC} \; ; \quad \frac{\partial v_2/v_1}{\partial g} = \frac{1}{sC}$$

used on p. 393 in a hidden way. See G. Temes & La Patra (book on synthesis & analysis)

Passive
RC circuits
do from LC circuit

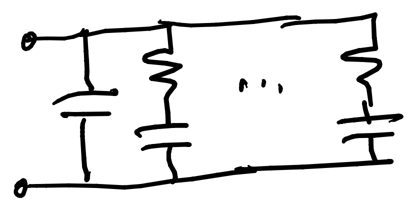
$$y_{LC}(s) = \sum_{i=0}^{m} \frac{k_i s}{s^2+\omega_i^2} + Cs \Rightarrow$$



2nd Foster

go to RC $\Rightarrow$ let L's $\Rightarrow$ R's

$$y_{i_{LC}} = \frac{k_i s}{s^2+\omega_i^2} = \frac{1}{\frac{s}{k_i} + \frac{\omega_i^2}{k_i s}}$$



2nd Foster

$$y_{i_{RC}} = \frac{1}{\frac{1}{k_i} + \frac{\omega_i^2}{k_i s}} \Rightarrow y_{RC} = \sum_{i=0}^{m} \frac{k_i s}{s+\omega_i^2} + Cs \Rightarrow \text{all poles are on } -\sigma \text{ axis}$$

look at $\dfrac{y_{RC}}{s} = \sum_{i=0}^{\infty} \dfrac{k_i}{s+\omega_i^2} + C$ is a partial fraction expansion with residues >0