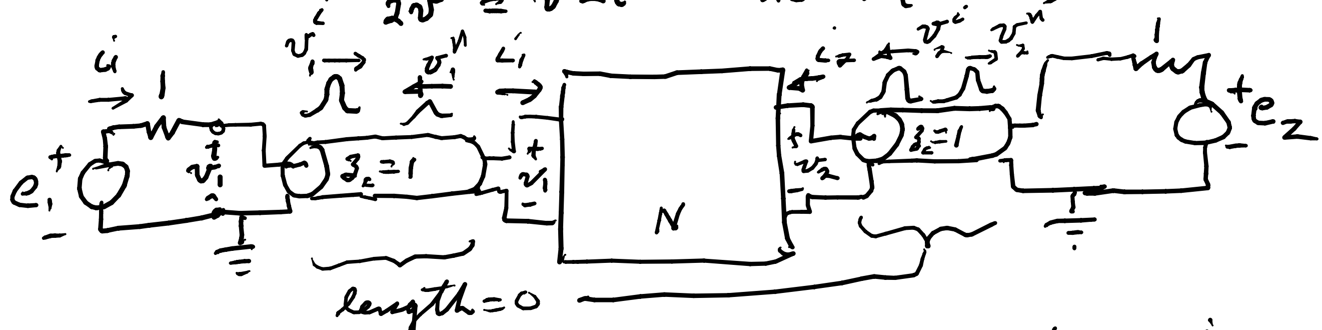
EE 610
10/18/11

Note Title 10/18/2011

S matrix $\quad 2v^i = v + i \qquad 2v = 2(v^i + v^n)$

$2v^n = v - i \qquad 2i = 2(v^i - v^n)$



$$v^n = S v^i \Rightarrow \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = \begin{bmatrix} v_1 + i_1 \\ v_2 + i_2 \end{bmatrix} = \begin{bmatrix} 2v_1^i \\ 2v_2^i \end{bmatrix} = 2v^i$$

$$\text{if } e_2 = 0 \Rightarrow v_2 = -i_2 \Rightarrow v_2^i \equiv 0$$

$$2v_2^n = v_2 - i_2 = 2v_2 \Rightarrow v_2 = v_2^n$$

$$v^n = \begin{bmatrix} v_1^n \\ v_2^n \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} v_1^i \\ v_2^i \end{bmatrix} \quad \text{for } e_2 = 0 \Rightarrow v_2^i = 0$$

$$\Rightarrow \underset{\overset{\shortparallel}{v_2}}{v_2^n} = S_{21} v_1^i = S_{21} \tfrac{1}{2} e_1$$

$$S_{21} = \frac{2v_2}{e_1} \Rightarrow S_{21} \text{ is the loaded voltage transfer function}$$

also $\quad v^n = S v^i \Rightarrow \frac{1}{2}(v - i) = S \frac{1}{2}(v + i)$

$$\Rightarrow v - Sv = i + Si =$$

$$(1_n - S)v = (1_n + S)i$$

but $\quad i = Y v \Rightarrow i = (1_n + S)^{-1}(1_n - S) \cdot v$

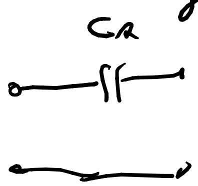
$$Y = (1_n + S)^{-1}(1_n - S) \underset{?}{=} (1_n - S)(1_n + S)^{-1}$$

$$\times (1_n + S) \text{ on left} \Rightarrow (1_n + S)(1_n + S)^{-1}(1_n - S) = 1_n - S \overset{?}{=} (1_n + S)(1_n - S)(1_n + S)^{-1} \text{ to see}$$

$$\text{on right} \Rightarrow (1_n - S)(1_n + S) \overset{?}{=} (1_n + S)(1_n - S)$$

$$1_n + S - S - SS \overset{?}{=} 1_n - S + S - SS$$

as true can go backwards if $(1_m+S)^{-1}$ exists

Ex: 

$\Rightarrow Y = \begin{bmatrix} Ca & -Ca \\ -Ca & +Ca \end{bmatrix} = Ca \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

need S as a function

$$(1_m+S)Y = (1_m - S) \Rightarrow Y + SY = 1_m - S$$

$$(1_m + Y)S = 1_m - S$$

$$Y - 1_m = -SY - S = -S(1_m + Y)$$

$$S = (1_m - Y)(1_m + Y)^{-1}$$

$$S = \left\{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - Ca\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\right\}\left\{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + Ca\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\right\}^{-1}$$

$$= \begin{bmatrix} 1-Ca & Ca \\ Ca & 1-Ca \end{bmatrix}\begin{bmatrix} 1+Ca & -Ca \\ -Ca & 1+Ca \end{bmatrix}^{-1}; \quad \det(1+Y) = 1+2Ca$$

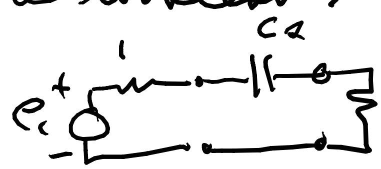
$$= \begin{bmatrix} 1-Ca & Ca \\ Ca & 1-Ca \end{bmatrix} \frac{1}{1+2Ca}\begin{bmatrix} 1+Ca & Ca \\ Ca & 1+Ca \end{bmatrix} =$$

$$S_{11} = \frac{1 - C^2a^2 + C^2a^2}{1+2Ca} = \frac{1}{1+2Ca}, \quad S_{12} = \frac{(1-Ca)Ca + (1+Ca)Ca}{1+2Ca} = \frac{2Ca}{1+2Ca}$$

$$S = \frac{1}{1+2Ca}\begin{bmatrix} 1 & 2Ca \\ 2Ca & 1 \end{bmatrix}$$

we know $Y(a) = -Y^T(-a)$ for a lossless Y

$$\Rightarrow S(a)S^T(-a) = 1_m$$



$v_2 = S_{21} \cdot e_1$
$= \frac{\cancel{3}\,\underline{2}\,Ca}{1+2Ca}$

Lossless $Y(a) = -Y(-a)^T = -Y_*^T$

$$\frac{v_2}{e_1} = \frac{1}{2 + \frac{1}{Ca}} = \frac{Ca}{2Ca+1} = \frac{1}{2}S_{21}$$

$$(1_m+S)^{-1}(1_m-S) = -\left[(1_m+S_*)^{-1}(1_m-S_*)\right]^T$$

$$= (-1_m + S_*)^T(1_m + S_*)^{-T}$$

$$(1_m - S)(1_m + S_*)^T = (1_m + S)(-1_m + S_*)^T$$

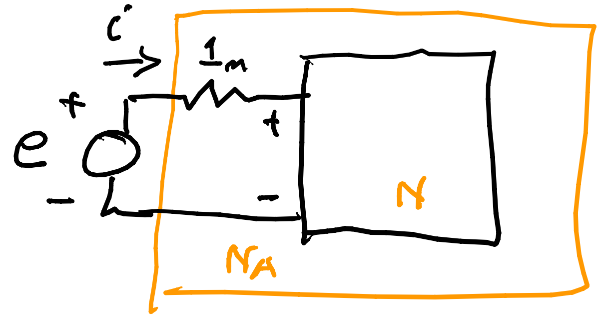
$$(1_m - S)(1_m + S_*^T) = (1_m + S)(-1_m + S_*^T)$$

$$1 - \cancel{S} + \cancel{S_*^T} - S S_*^T = -1_m - \cancel{S} + \cancel{S_*^T} + S S_*^T$$

$$2 1_m = 2 S S_*^T \Rightarrow S \cdot S_*^T = 1_m$$

$$\Rightarrow S^{-1} = S_*^T \Rightarrow$$ lossless condition

for passive S has no poles in $\sigma \geq 0$:



augmented

$$e = 2v^i = v + i; \quad i = Y_A \cdot e = Y_A(v+i) \Rightarrow Y_A v = (1_m - Y_A) i$$

$$2Y_A v^i = Y_A e = Y_A v + Y_A i = 1_m i - Y_A i + Y_A i = i$$

$$2 v^r = v - i$$

$$2Y_A v^r = Y_A v - Y_A i = (1_m i - Y_A i) - Y_A i = (1_m - 2Y_A) i$$

$$= (1_m - 2Y_A)\cdot 2Y_A v^i$$

$$= (2Y_A - 4Y_A^2) v^i = 2Y_A(1_m - 2Y_A) v^i$$

$$v^r = (1_m - 2Y_A) v^i \Rightarrow S = 1_m - 2Y_A$$

this has loss in Y_A (as had series resistors)

$\therefore$ S has no poles in $\sigma \geq 0$

here S will exist for "any" passive circuit

$$A \cdot v = B \cdot i \Rightarrow A(v^i + v^r) = B\cdot(v^i - v^r)$$

$$(A - B) v^i = (-B - A) v^r \Rightarrow v^r = (B+A)^{-1}(B-A) v^i$$

$$\Rightarrow S = (B+A)^{-1}(B-A)$$ for Z, $A = 1_m$, $B = Z$

$$S = (1_m + Z)^{-1}(1_m - Z)$$

$$Y_A v = (1_m - Y_A) i \Rightarrow A = Y_A, \; B = 1_m - Y_A$$

$$S = (1_m - Y_A + Y_A)^{-1}(1_m - Y_A - Y_A) = 1_m - 2Y_A$$

$$S = \frac{1}{1+2Ca}\begin{bmatrix} 1 & 2Ca \\ 2Ca & 1 \end{bmatrix} \quad \text{form } S S_*^T \overset{?}{=} 1_2$$

$$\frac{1}{1+2Ca}\begin{bmatrix} 1 & 2Ca \\ 2Ca & 1 \end{bmatrix}\begin{bmatrix} 1 & -2Ca \\ -2Ca & 1 \end{bmatrix}\frac{1}{1-2Ca} = \frac{1}{1-4C^2a^2}\begin{bmatrix} 1-4C^2a^2 & 0 \\ 0 & 1-4C^2a^2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Condition for passive, rational; BR = bounded real

1) S has real coefficients

2) S no poles in $\sigma \geq 0$

3 $1_n - S(j\omega)S^T(-j\omega)$ is positive semi definite

$$\Rightarrow \quad v^{i\,T}(j\omega)\left[1_n - S(j\omega)S^T(-j\omega)\right]v^i(j\omega) \geq 0$$

for all $v^i(j\omega)$

next $\Rightarrow$ sensitivity via adjoint