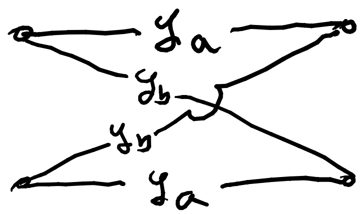
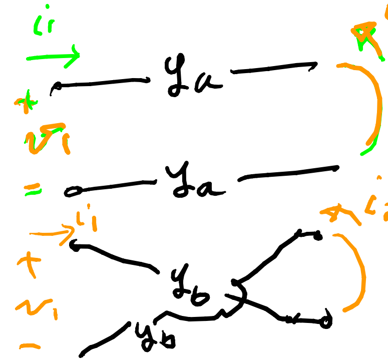
EE 610
10/11/11

Lattice: (symmetric)



$\Rightarrow$

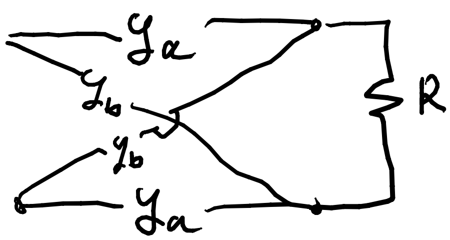


$\left.\frac{i_2}{v_1}\right| = y_{21}$, $v_2 = 0$

$$Y_a = \begin{bmatrix} y_a/2 & -y_a/2 \\ -y_a/2 & y_a/2 \end{bmatrix}$$

$$Y_b = \begin{bmatrix} y_b/2 & +y_b/2 \\ y_b/2 & y_b/2 \end{bmatrix} \Rightarrow Y = \frac{1}{2}\begin{bmatrix} y_b + y_a & y_b - y_a \\ y_b - y_a & y_b + y_a \end{bmatrix}$$

$y_{in} \rightarrow$



$$y_{in} = y_{11} - y_{12}\frac{1}{y_{22}+G}y_{21}$$

$$= \frac{\Delta_y + y_{11}G}{y_{22}+G} = G\left(\frac{y_{11} + \frac{\Delta}{G}y}{y_{22}+G}\right)$$

$$\Delta = \left(\frac{y_b + y_a}{2}\right)^2 - \left(\frac{y_b - y_a}{2}\right)^2$$

$$= \left(y_b^2 + y_a^2 + 2y_a y_b - y_b^2 - y_a^2 + 2y_a y_b\right)_4 = y_a y_b$$

$$y_{in} = G\left(\frac{y_{11} + y_a y_b/G}{y_{11} + G}\right)$$

choose $y_a y_b = G^2 \Rightarrow y_{in} = G$

gives the constant R lattice

$\frac{y_a}{G} = \frac{G}{y_b} \Rightarrow$ duals for the two arm types

Look at the voltage transfer ratio: $\underbrace{-Gv_2 = i_2 = y_{21}v_1 + y_{22}v_2}_{v_2/v_1}$

$$\frac{v_2}{v_1} = \frac{-y_{21}}{y_{22}+G} = \frac{-(y_b - y_a)/2}{\left(\frac{y_b + y_a}{2}\right) + G}$$

$$\frac{v_2}{v_1} = \frac{y_a - y_b}{y_a + y_b + 2G} = \frac{G^2/y_b - y_b}{G^2/y_b + y_b + 2G} = \frac{G^2 - y_b^2}{y_b^2 + 2Gy_b + G^2}$$

$$= \frac{(G - y_b)(G + y_b)}{(G + y_b)^2} = \frac{G - y_b}{G + y_b} = \frac{1 - y_b/G}{1 + y_b/G} = A_v$$

$$1 - y_b/G = A_v + (y_b/G)A_v \Rightarrow (1 + A_v)\frac{y_b}{G} = 1 - A_v$$

$$\frac{y_b}{G} = \frac{1 - A_v}{1 + A_v} \; ; \; Re\left(\frac{y_b}{G}\right) = Re\left(\frac{1 - A_v}{1 + A_v}\right) \text{ in } \sigma > 0$$

$$= \frac{Re\left((1 - A_v)(1 + A_v)^*\right)}{|(1 + A_v)|^2}$$

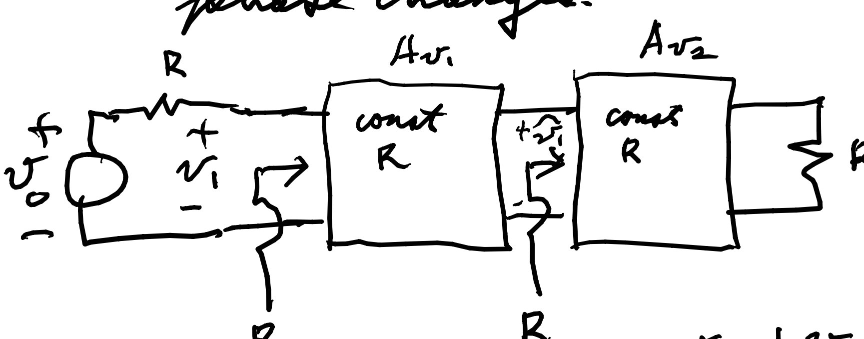
$$= \frac{Re\left(1 - A_v + A_v^* - |A_v|^2\right)}{|(1 + A_v)|^2}$$

$$= \frac{Re\left(1 - |A_v|^2\right)}{|(1 + A_v)|^2}$$

$$= \frac{1 - |A_v|^2}{|(1 + A_v)|^2} \quad \text{desire} \geq 0 \text{ in } \sigma > 0$$

for y_b/G to be PR

for y_b/G to be PR we need $|A_v| \leq 1$ in $\sigma \geq 0$ & then we can synthesize y_b (and y_a via the dual) by a passive. If lossless then can use Cauer or Foster forms. y_b will be lossless if $|A_v(j\omega)| = 1$. This will be an all-pass circuit $\Rightarrow$ gives phase changes.



$$v_2 = A_{v_2} \cdot \hat{v}_1 = A_{v_2} \cdot A_{v_1} \cdot v_1$$

$$A_v = A_{v_2} \cdot A_{v_1}$$

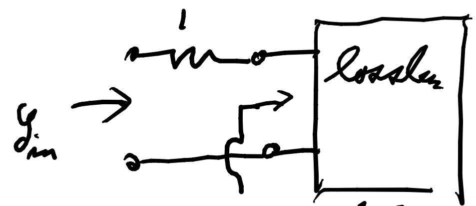
$$v_1 = \frac{1}{2} v_0$$

Look at Hurwitz polynomial

$P(s) = \sum_{i=1}^{\delta} a_i s^i$ (strictly Hurwitz if no zeros in $\sigma \geq 0$)

Hurwitz if all zeros are in $\sigma < 0$ & those on $\sigma = 0$, $= j\omega$ axis, are simple

$\Rightarrow$ stable circuit, characteristic "functions"



$\Rightarrow y_{in} = \dfrac{1 \times y}{1+y} \Rightarrow$ denominator is Hurwitz

$y = \dfrac{Od \cdot P(s)}{Ev \cdot P(s)} = \dfrac{N}{D}$ $\qquad = \dfrac{N/D}{1+N/D} = \dfrac{N}{N+D} = \dfrac{N}{P(s)}$ Hurwitz

$= \dfrac{P(s) - P(-s)}{P(s)+P(-s)}$ given $P(s)$ a Hurwitz polynomial

$$\text{Given } P(s) = \underbrace{\left(\frac{P(s)+P(-s)}{2}\right)}_{Ev\,P} + \underbrace{\left(\frac{P(s)-P(-s)}{2}\right)}_{Od\,P}$$

then divide $\dfrac{Od\,P}{Ev\,P}$ = a lossless admittance (reactance function & can synthesize)

Richards' function

p.261 $\qquad R(s) = \dfrac{k Z(s) - s Z(k)}{k Z(k) - s Z(s)}$ eq. (8.6-1)

is PR if $Z(s)$ is PR & k is real & > 0

note a pole seems to be at $s = k$; $kZ(k) - kZ(k) = 0$ but at $s = k$ also have 0; $kZ(k) - kZ(k)$ which cancels. Note δ seems to go up by 1 but we cancelled one pole & 0 $\Rightarrow \delta R \leq \delta Z$)

$\therefore$ desire to cancel another pole with another 0.

We choose k properly.

df $\quad Ev\, z(s) = \frac{z(s)+z(-s)}{2}$, choose k, $Ev\, z(k) = 0$

$$z(k) = -z(-k)$$

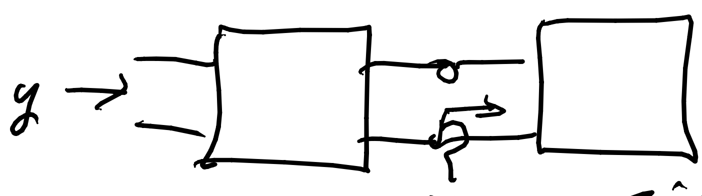
$$R(s) = \frac{k z(s) - s z(k)}{k z(k) - s z(s)}\Bigg|_{s=-k} \Rightarrow \frac{k z(-k) - (-k) z(k)}{k z(k) - (-k) z(-k)} = \frac{k\overbrace{(z(-k)+z(k))}^{=0}}{\underbrace{k(z(k)+z(-k))}_{=0}}$$

∴ there is no another pole & zero cancellation at $s = -k \Rightarrow \delta(R(s)) = \delta(z(s)) - 1$

∴ keeps doing until $\delta(R_{\dots}(s)) = 0 \Rightarrow$ resistor (open or short)

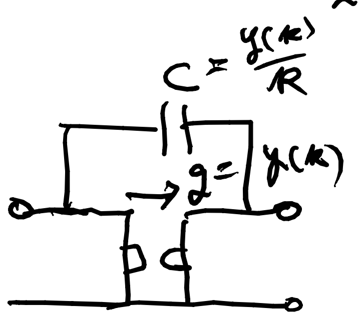
∴ for lossless we can choose any k as the $y(s)$ or $z(s)$ has its even part $\equiv 0$ for all s.

∴ look for a circuit to given $y_{load}(s)$ = a Richards' function to synthesize a $y = y_{in}$



y_L = a Richard's function

so $\delta(y_L) = \delta(y) - 1$



$$Y = \begin{bmatrix} sC & -sC+g \\ -sC-g & sC \end{bmatrix}; \quad \Delta Y = g^2$$

$$y_{in} = y_{11} - y_{12}\frac{1}{y_{22}+y_L}y_{21} = \frac{\Delta Y + y_{11}y_L}{y_{22}+y_L} = \frac{g^2 + sC\, y_L}{sC + y_L}$$

$$y_{in}\, sC + y_{in}\, y_L = g^2 + sC\, y_L$$

$$y_L = \frac{g^2 - sC\, y_{in}}{y_{in} - sC} = \frac{g^2 - sC\, y(s)}{y(s) - sC} = g^2\left(\frac{1 - \frac{sC}{g^2}y(s)}{y(s) - sC}\right)$$

$$\frac{y_L}{g} = \left(1 - \frac{sC}{g^2}y(s)\right) \Big/ \left(y(s)/g - sC/g\right)$$

In Richards' function replace $z(s) = 1/y(s)$

$$R(s) = \frac{k\,y(k) - s\,y(s)}{k\,y(s) - s\,y(k)} = \frac{y(k) \times k\left(1 - \frac{s}{k} \cdot \frac{y(s)}{y(k)}\right)}{k\left(y(s) - \frac{s}{k}\,y(k)\right)} =$$

$$= \left(\frac{1 - \frac{s}{k\,y(k)} \cdot y(s)}{y(s)/y(k) - \frac{s}{k}}\right)$$

$$\frac{s}{k\,y(k)} = \frac{sC}{g^2} \quad \& \quad \frac{s}{k} = \frac{sC}{g}$$

$$\Rightarrow g = y(k), \quad C = \frac{g}{k} = \frac{y(k)}{k}$$

$$= \frac{y_L}{g} = \left(\frac{1 - \frac{sC}{g^2}\,y(s)}{\frac{y(s)}{g} - \frac{sC}{g}}\right)$$

$\Rightarrow$ form $y_L(s) = g\,R(s) = y(k)\left(\dfrac{k\,y(k) - s\,y(s)}{k\,y(s) - s\,y(k)}\right)$