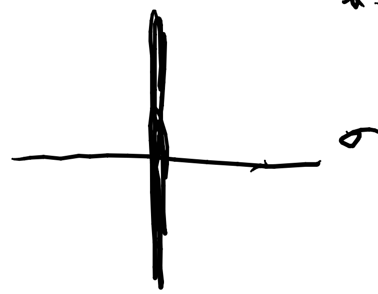
EE610
10/06/11

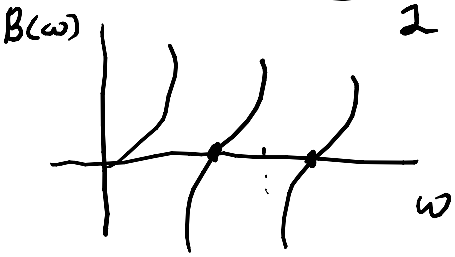
Note Title 10/6/2011

Reactance function $f(s)$; rational + passive + $\frac{f(s)+f(-s)}{2} = 0$ $\quad s=\sigma+j\omega$

$P_{in}(j\omega) = 0$ for all ω

$\Rightarrow \frac{f(s)+f(-s)}{2} = 0 \Rightarrow$



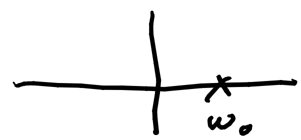


@ a pole $\approx \frac{ks}{s^2+\omega_0^2}$

$$E_v\left(\frac{ks}{s^2+\omega_0}\right) = \frac{ks}{s^2+\omega_0^2} + \frac{k(-s)}{(-s)^2+\omega_0^2} \equiv 0$$

if passive, no poles in $\sigma > 0$

non passive $\frac{ks}{-s^2+\omega_0^2}$ has $E_v \equiv 0$



for passive no poles in $\sigma > 0$

Ex: $y(s) = \frac{(s^2+1)(s^2+4)}{s(s^2+3)}$; $z(s) = \frac{s(s^2+3)}{(s^2+1)(s^2+4)}$ no pole at ∞

$$= \frac{s^3+3s}{s^4+5s^2+4}$$

$$y(s) = \frac{s^4+5s^2+4}{s^3+3s}$$

1st Cauer, remove poles at ∞ (if none, there is a 0 at ∞ so turn it over & there is a pole)

divide highest power of den into num

$$y(s) = s + \frac{2s^2+4}{s^3+3s} = y_c + y_{rem}$$

$\underbrace{\frac{2s^2+4}{s^3+3s}}_{y_{rem}}$ no pole at ∞

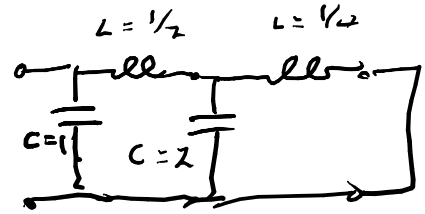
$$\begin{array}{r|l} & s \\ s^3+3s & s^4+5s^2+4 \\ & s^4+3s^2 \\ \hline & 2s^2+4 \end{array}$$

$$\begin{array}{r|l} & \frac{1}{2}s \\ 2s^2+4 & s^3+3s \\ & s^3+2s \\ \hline & s \end{array}$$

$$\begin{array}{r|l} & 2s \\ s & 2s^2+4 \end{array} \qquad \begin{array}{r|l} & \frac{1}{4}s \\ 4 & s \end{array}$$

$z_{rem} = \frac{1}{y_{rem}} = \frac{s^3+3s}{2s^2+4}$ has a pole at ∞

$$z(s) = \cfrac{1}{\underbrace{s}_{y_1} + \underbrace{\cfrac{1}{\frac{1}{2}s + \cfrac{1}{2s + \cfrac{1}{\frac{1}{4}s}}}}_{y_2}} \quad \begin{matrix} \leftarrow z \\ \leftarrow y \\ \leftarrow z \\ \leftarrow y \\ \leftarrow z \end{matrix}$$



continued fraction expansion about $s = \infty$

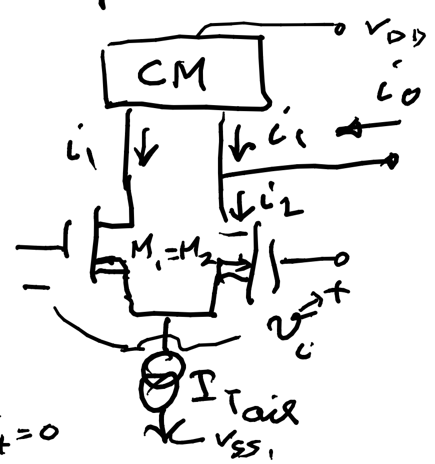
look at $$y(s) = \frac{s(s^2+4)}{(s+1)(s^2+9)} = \frac{k_1}{(s+j3)} + \frac{k_2}{(s-j3)} + \frac{k_3}{s+1} + 1$$

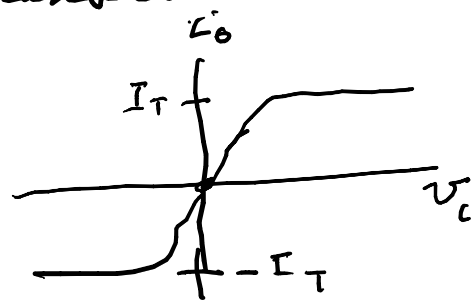
$$k_1 = \left.\frac{s(s^2+4)\cancel{(s+j3)}}{(s+1)\cancel{(s+j3)}(s-j3)}\right|_{s=-j3} = k_1 + \left[k_2\frac{(s+j3)}{(s-j3)} + k_3\frac{(s+j3)}{s+1} + (s+j3) \right]_{s=-j3}$$

$$= \frac{-j3(-9+4)}{-j3+1} = \frac{+j15}{1-j3} = k_1 \neq \text{real}$$

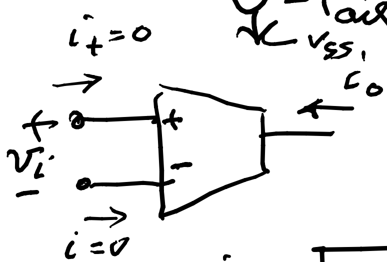
$\therefore$ this $y(s)$ is not PR

OTA = operational transconductance amplifier

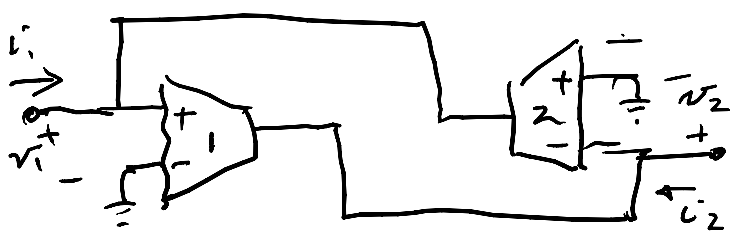




linearize @ $v_i = 0 \Rightarrow g_m$



$$Y = \begin{bmatrix} 0 & 0 \\ g_m & 0 \end{bmatrix}$$



$$Y = \begin{bmatrix} 0 & -g_{m_2} \\ g_{m_1} & 0 \end{bmatrix}$$

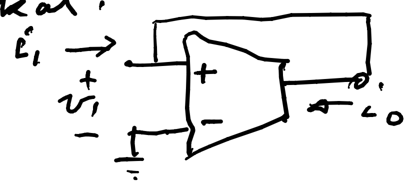
look at $\dfrac{V^{T*} I + I^{T*} V}{2}$; $V = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$, $\begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$

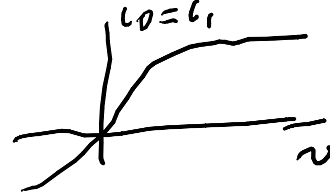
$$\frac{V^{T*}YV + V^{T*}Y^{T*}V}{2} = \frac{V^{T*}(Y+Y^{T*})V}{2} \quad \text{desire} \geq 0 \text{ for passive}$$

$$Y + Y^{T*} = Y + Y^{T} = \begin{bmatrix} 0 & -g_{m_2} \\ g_{m_1} & 0 \end{bmatrix} + \begin{bmatrix} 0 & g_{m_1} \\ -g_{m_2} & 0 \end{bmatrix} = \begin{bmatrix} 0 & g_{m_1} - g_{m_2} \\ g_{m_1} - g_{m_2} & 0 \end{bmatrix}$$

$\therefore$ it is passive if & only if $g_{m_1} = g_{m_2} \Rightarrow$ lossless

look at:

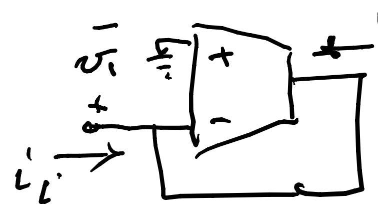


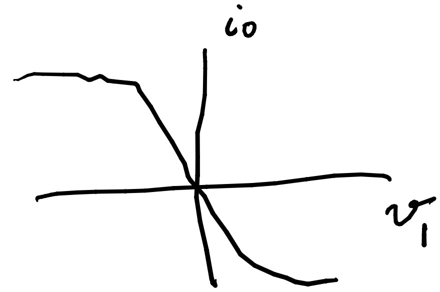


positive

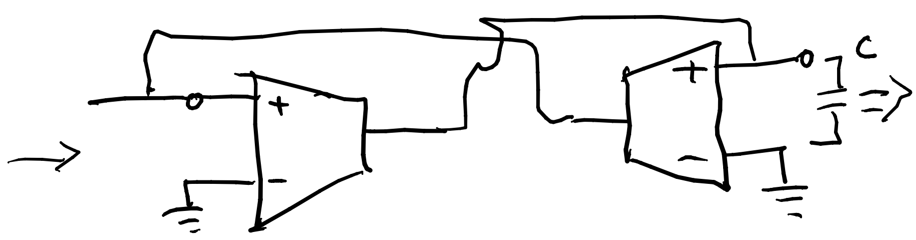
$$[v_1 \; v_2]\begin{bmatrix} 0 & g \\ g & 0 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = [g v_2 \; g v_1]\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = 2g\,v_1 v_2$$

is < 0 if $v_1 = -v_2$





$\Rightarrow$ a negative resistor



$$\Rightarrow Y = \begin{bmatrix} 0 & g_{m_2} \\ g_{m_1} & 0 \end{bmatrix}$$

$$y_{in} = y_{11} - y_{12}\frac{1}{y_{22}+y_{\ell}} \cdot y_{21} = -g_{m_2}\frac{1}{sC} \cdot g_{m_1} =$$

$$= \frac{-g_{m_1} g_{m_2}}{sC} \Rightarrow L_{eq} = \frac{-C}{g_{m_1} g_{m_2}} \Rightarrow \text{negative inductor}$$

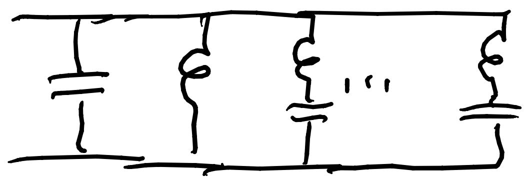
$$= \frac{1}{L_{eq}\, s} \quad \text{residue @ the pole at } s = 0 \text{ is } -\frac{g_{m_1} g_{m_2}}{C}$$

but if g_{m_1} & g_{m_2} have opposite signs the residue is positive if $C > 0 \Rightarrow y_{in}$ is PR

Ex: $C = 10^{-9}$ fd, $g_{m_1} = 10^{-3} = -g_{m_2}$

$$L_{eq} = \frac{10^{-9}}{10^{-3} \times 10^{-3}} = 10^{-3} \text{ hy} = 1 \text{ mhy}$$

LC circuit $y_{LC} = \sum \frac{2k_i s}{s^2 + \omega_i^2} + \frac{k_0}{s} + k_\infty s$

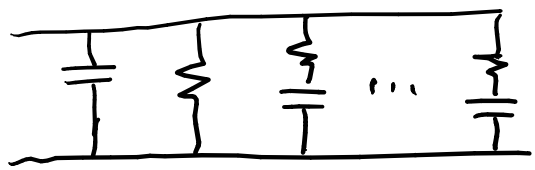


$$y_{i_{LC}} = \frac{1}{L_i s + \frac{1}{sC_i}}$$

$$\Downarrow$$

$$y_{i_{RC}} = \frac{1}{r_i + \frac{1}{sC_i}} = \frac{sC_i}{1 + s r_i C_i}$$

can replace L by R & get all RC circuits have this 2nd Foster form



$\Rightarrow y_{RC} = k_\infty s + r_0 + \sum \frac{g_i s}{s + g_i/C_i}$

has all poles on negative σ axis

$$\frac{y_{RC}(s)}{s} = k_\infty + \frac{r_0}{s} + \sum \frac{g_i}{s + g_i/C_i}$$

here the g_i are residues & must be > 0 for a passive RC circuit

poles & zeros will alternate on $-\sigma$ axis