EE610

10/04/11

Note Title 10/4/2011

PR scalar $y(s)$

1) coefficients real — real elements
2) no poles in open RHP — stable
3) $\mathrm{Re}\, y(s) \geq 0$ in the open RHP — passive

SSS = sinusoidal steady state; $s = j\omega$

average power $\mathrm{Re}\, V^* I \underset{\text{passive}}{\geq 0}$; $\mathrm{Re}(V^* I) = \dfrac{V^* I + V I^*}{2}$

if $I = Y(j\omega) V \Rightarrow 2\,\mathrm{Re}(V^* I) = V^* Y(j\omega) V + V Y^*(j\omega) V^*$

(scalar) $= V^* V (Y(j\omega) + Y^*(j\omega))$

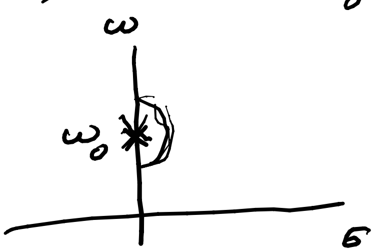
$= |V|^2 (Y(j\omega) + Y(-j\omega))$ if PR

$= |V|^2 (Y(s) + Y(-s)) \Big|_{s=j\omega}$

$\mathrm{Re}\, y(s) = \dfrac{y(s) + y^*(s)}{2}$, $\mathrm{Ev}\, y(s) = \dfrac{y(s) + y(-s)}{2}$ = analytic in s

if $\mathrm{Re}\, V^* I = 0$ in SSS we call "lossless"

$\Rightarrow$ no poles of $y(s)$ outside of $j\omega$ axis if lossless $y(s)$



near such a pole $y(s) \approx \dfrac{k}{(s - j\omega_0)^n}$

desire to know $n = 1$, k real

$$y(s) \approx \frac{k}{(s - j\omega_0)^n} = \frac{|k| e^{j\angle k}}{|s - j\omega_0|^n e^{n\angle(s - j\omega)}} = \frac{|k|}{|s - j\omega_0|^n} e^{j(\angle k - n\angle(s - j\omega_0))}$$

$$\mathrm{Re}\, y(s) \Big| = \left| \frac{k}{s - j\omega_0} \right|^n \cos\big(\underbrace{\angle k - n\angle(s - j\omega_0)}\big)$$ must be ≥ 0 for $y(s)$ to be PR

s in $\sigma > 0$ on a small circle

$\Rightarrow -\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$

requires $\angle k = 0$, $n = 1$

$\Rightarrow$ poles on $j\omega$ axis are simple $(n = 1)$ with positive residue $(\angle k = 0)$

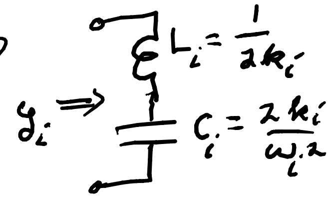
If $y(s)$ is LPR (= lossless PR) then all poles are on $j\omega$ axis & simple with positive residue (also any pole comes with its conjugate)

$$y(s) = \sum_{i=1}^{m}\left(\frac{k_i}{s-j\omega_i} + \frac{k_i}{s+j\omega_i}\right) + \frac{k_0}{s} + k_\infty s$$ partial fraction expansion

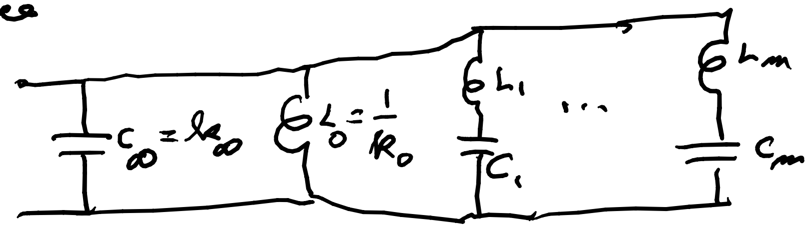
$$= \sum_{i=1}^{m} \frac{k_i[2s]}{s^2+\omega_i^2} + \frac{k_0}{s} + k_\infty s = -y(-s)$$

can synthesize: $$y_i(s) = \frac{2k_i s}{s^2+\omega_i^2} = \frac{1}{\frac{s}{2k_i} + \frac{\omega_i^2}{2k_i s}} = \frac{1}{z_i(s)}$$

$$z_i(s) = \frac{s}{2k_i} + \frac{\omega_i^2}{2k_i s} \Rightarrow$$

$y_i \Rightarrow$ 

Synthesis of a lossless 1-port PR admittance

$y(s) \Rightarrow$ 

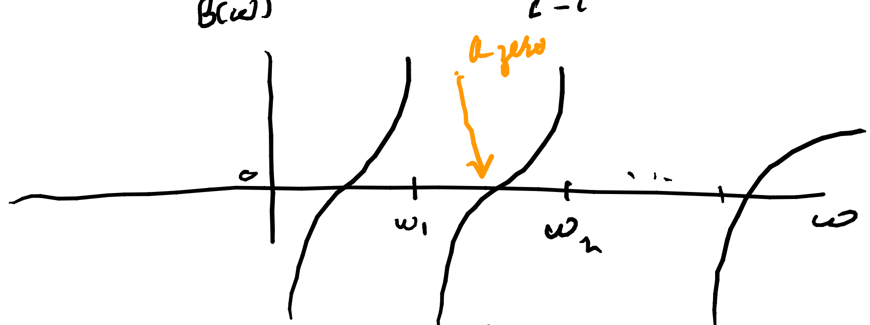
minimal as $\delta[y] = 2m+2$ is number of reactive elements (L's & C's)

Look on $s = j\omega$ at

$$y(j\omega) = \sum_{i=1}^{m} \frac{2k_i(j\omega)}{-\omega^2+\omega_i^2} + \frac{k_0}{j\omega} + k_\infty j\omega = jB(\omega)$$

$$\frac{dB(j\omega)}{d\omega} = k_\infty + \frac{k_0}{\omega^2} + \sum_{i=1}^{m} \frac{2k_i}{(-\omega^2+\omega_i^2)} - \frac{2k_i\omega(-2\omega)}{(-\omega^2+\omega_i^2)^2}$$

$$= k_\infty + \frac{k_0}{\omega^2} + \sum_{i=1}^{m} \frac{2k_i(\omega^2+\omega_i^2)}{(-\omega^2+\omega_i^2)^2} \geq 0$$



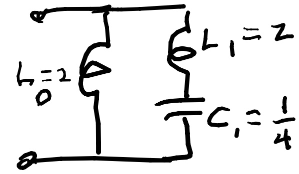
as $\frac{dB}{d\omega} > 0$

$\therefore$ poles and zeros alternate on $j\omega$ axis for a lossless PR $y(s)$

same for $Z(s)$

Ex: $y(s) = \frac{1}{s}\frac{(s^2+1)}{(s^2+2)}$ is PR & lossless, $y(s) = -y(-s)$

$$= \frac{k_0}{s} + \frac{2k_1 s}{s^2+2} \Rightarrow$$



2nd Foster

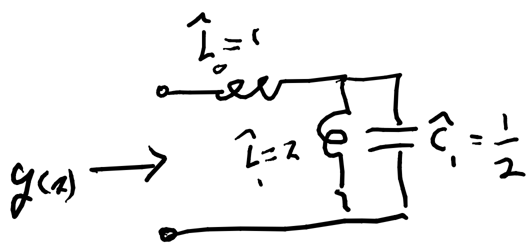
$$\times s \quad \frac{s}{s}\left(\frac{s^2+1}{s^2+2}\right)\Bigg|_{s=0} = k_0 + \underbrace{\frac{2k_1 s^2}{s^2+2}}_{=0 @ s=0}\Bigg|_{s=0} \Rightarrow k_0 = 1/2$$

$$\times \frac{s^2+2}{s} \quad \frac{s^2+2}{s^2}\cdot\frac{s^2+1}{s^2+2}\Bigg|_{s^2=-2} = \underbrace{\frac{k_0(s^2+2)}{s^2}}_{=0 @ s^2=-2} + 2k_1 \Rightarrow \frac{s^2+1}{s^2}\Bigg|_{s^2=-2} = \frac{-2+1}{-2} = \frac{+1}{2} = 2k_1$$

$$y(s) = \frac{1/2}{s} + \frac{\frac{1}{2}s}{s^2+2} = \frac{\frac{1}{2}s^2+1+\frac{1}{2}s^2}{s(s^2+2)} = \frac{s^2+1}{s(s^2+2)}$$

$$Z(s) = \frac{s(s^2+2)}{s^2+1} = s + \frac{2ks}{s^2+1} \Rightarrow 2k = s^2+2\Big|_{s^2=-1} = 1$$

$$= s + \frac{2s}{s^2+1}$$

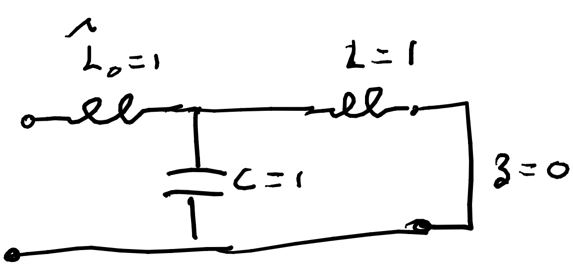


1st Foster

$Z(s) \Rightarrow$

$$\begin{array}{l}
\quad\quad\quad\quad\ s \leftarrow z_0 \\
(s^2+1)\overline{)\,s^3+2s} \\
\quad\quad\quad\ \ \underline{s^3+s} \\
\quad\quad\quad\quad\quad s \\
\quad\quad\quad\ \ \overline{s^2+1}
\end{array}
\qquad
\begin{array}{l}
\quad\ s \leftarrow y_0 \\
s\overline{)\,s^2+1} \\
\quad\ \underline{s^2} \\
\quad\quad 1
\end{array}
\qquad
\begin{array}{l}
\quad s \leftarrow z \\
1\overline{)\,s} \\
\quad \underline{s} \\
\quad 0 \leftarrow
\end{array}$$

continued fraction about $s=\infty$



gives a ladder

1st Cauer

2nd Cauer $\Rightarrow$ continued fraction expansion about $s=0$
i.e. lowest power of s into lowest power of s

$$y(s) = \frac{1}{s}\frac{(s^2+1)}{(s^2+2)} = \frac{1+s^2}{2s+s^3}$$

$$\frac{1}{2s} \leftarrow y$$

$$2s+s^3 \,\big)\, 1 + s^2$$

$$1 + s^2/2 \qquad \frac{4}{s} \leftarrow z$$

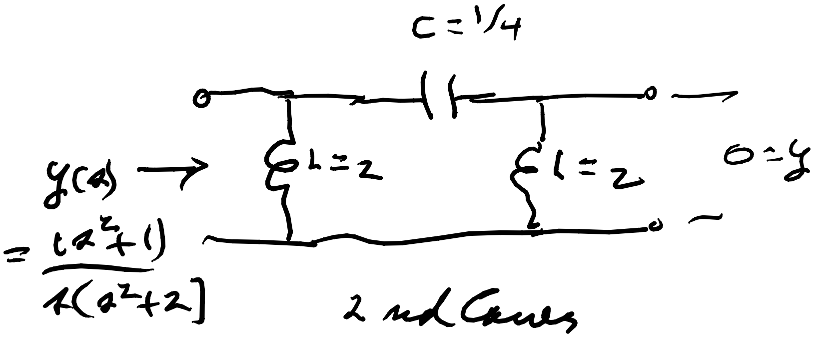
$$s^2/2 \,\big)\, 2s + s^3$$

$$2s \qquad \frac{1}{2s} \leftarrow y$$

$$s^3 \,\big)\, s^2/2$$

$$s^2/2$$

$$0 \quad s$$



2 nd Cauer

∴ have 4 canonical lossless circuits for an LPR