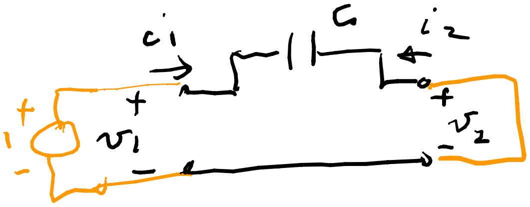
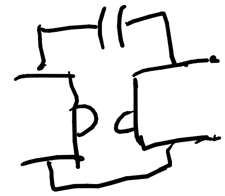
EE610
09/29/11

Note Title 9/29/2011



$i_1 = y_{11} v_1 + y_{12} v_2$

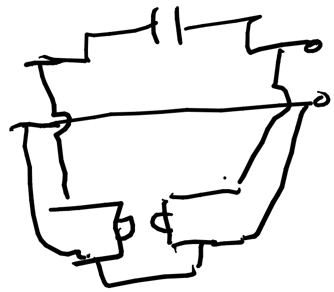


$i = y_c \cdot 1$

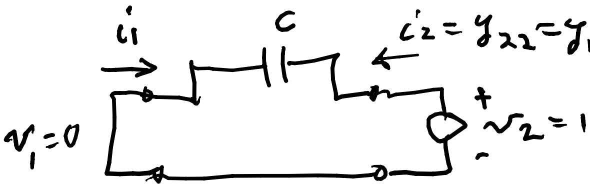
$y_{11} = sC$

$Y = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{11} \end{bmatrix}$

$\equiv$



$y_{11} = \left.\dfrac{i_1}{v_1}\right|_{v_2=0}$, $y_{12} = \left.\dfrac{i_1}{v_2}\right|_{v_1=0} = -y_{11}$



$i_2 = y_{22} = y_{11} = -i_1$

$Y = \begin{bmatrix} sC & -sC \\ -sC & sC \end{bmatrix}$

y_{12}

$T(s) = \dfrac{1}{s+a}$, $a > 0$; $\mathcal{L}[f(t)] = \displaystyle\int_{-\infty}^{\infty} f(t) e^{-st} dt$

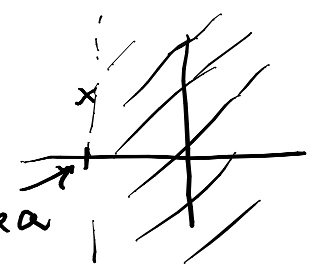
if $f(t) = e^{-at} 1(t)$, $1(t) = \begin{cases} 1 & \text{if } t > 0 \\ 0 & \text{if } t < 0 \end{cases}$

$\mathcal{L}[e^{-at} 1(t)] = \displaystyle\int_{-\infty}^{\infty} e^{-at} 1(t) e^{-st} dt = \int_0^{\infty} e^{-(s+a)t} dt$

$= \left.\dfrac{-1}{s+a} e^{-(s+a)t}\right|_{t=0}^{t=\infty} = \dfrac{1}{s+a} + \dfrac{-1}{s+a} e^{-(s+a)\infty}$

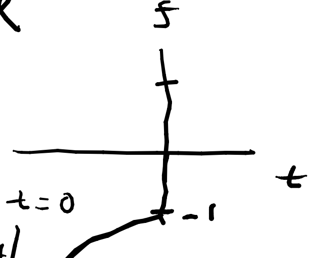
$s = \sigma + j\omega$

we require $\text{Re}(s+a) > 0 \Rightarrow \sigma > -\text{Re}\, a$



if $f(t) = -e^{-at} 1(-t)$ is not causal

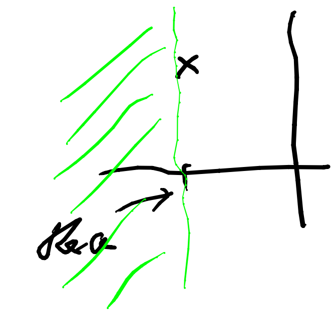
$\mathcal{L}[-e^{-at} 1(-t)] = \displaystyle\int_{-\infty}^{\infty} -e^{-at} 1(-t) e^{-st} dt$



$= -\displaystyle\int_{-\infty}^{0} e^{-(s+a)t} dt = \left.\dfrac{+1}{s+a} e^{-(s+a)t}\right|_{t=-\infty} = \dfrac{1}{s+a} - \dfrac{1}{s+a} e^{(s+a)\infty}$

needs $\text{Re}(s+a) < 0$

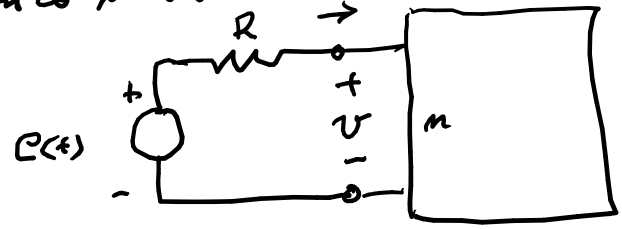
i.e. $T(s) = \frac{1}{s+a} \Rightarrow f(t) = -e^{-at}\,1(-t)$ if $\mathrm{Re}\, s < \mathrm{Re}\, a$



$\Rightarrow$ i.e. to go back to time from $T(s)$ we need the region of convergence in s

region of convergence

Return to passive



$e = Ri + v$

normalize $R = 1\,\Omega$

$$e^{T*}(\tau)\,e(\tau) = (i+v)^{T*}(i+v) = i^{T*}i + v^{T*}i + i^{T*}v + v^{T*}v$$

$$= i^{T*}i + 2\,\mathrm{Re}\, v^{T*}i + v^{T*}v$$

$$\int_{-\infty}^{t} e^{T*}(\tau)\,e(\tau)\,d\tau = \int_{-\infty}^{t} \sum_{i=1}^{n} |e_i|^2\, d\tau$$

assume finite for all t also for $t=\infty$

assume this to have e in L_2

$$= \int_{-\infty}^{t} \Big(i^{T*}i + \underbrace{2\,\mathrm{Re}\, v^{T*}i}_{\text{this is power}} + v^{T*}v \Big)\, d\tau \Rightarrow$$

if the circuit is passive v & i are L_2 functions

This means $\mathcal{L}[v]\Big|_{s=j\omega}$ & $\mathcal{L}[i]\Big|_{s=j\omega}$ exist

Look at Parseval's result:

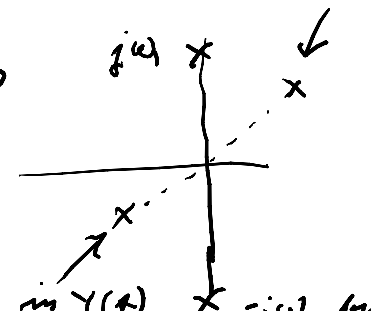
$$\int_{-\infty}^{\infty} F^{*T}(j\omega)\, G(j\omega)\, \frac{d\omega}{2\pi} = \int_{\infty}^{\infty} f^{*T}(\tau)\, g(\tau)\, d\tau$$

$\mathcal{L}[v]^{*T} \cdot \mathcal{L}[i]\Big|_{s=j\omega}$ $\qquad$ $\overset{\shortparallel}{v^{T*}\, i} \qquad \geq 0$

$= V^{*T} \cdot Y(s)\, V$ $\qquad$ $i^{T*}v$

add on $\quad I^{*T} V$

$$\int V^{*T}\left(Y(s) + Y^{T*}(s)\right)\Big|_{s=j\omega} V \frac{d\omega}{2\pi} = \int v^{T*} i + i^{T*} v \, dt \geq 0$$



for any constant vectors V

$\Rightarrow \quad V^{*T}\left(Y(s) + Y^{T}(-s)\right)\Big|_{s=j\omega} V$ needs to be ≥ 0

$\uparrow$

no poles in $\sigma > 0$

for $Y(s) = y(s)$

poles on $s = j\omega$ are simple with positive real residue (scalar)

$y(s) \approx \dfrac{k}{s+j\omega_1} + \dfrac{k}{s-j\omega_1} = \dfrac{2ks}{s^2+\omega_1^2}$ near the pole

near the pole $\quad y(s) + y(-s) \approx \dfrac{2ks}{s^2+\omega_1^2} + \dfrac{-2ks}{s^2+\omega_1^2} \approx 0$

here $\quad \dfrac{y(s)+y(-s)}{2} = \mathcal{E}v\, y(s)$

$\dfrac{y(s)+y^*(s)}{2} = \operatorname{Re} y(s) = \mathcal{E}v\, y(s)$ if $s = j\omega$ for $y(s)$ rational with real coefficients

then $\mathcal{E}v\, y(s)$ is rational

$$e^{-y(s)} = |e^{-y(s)}|$$
$$= |e^{-\operatorname{Re} y(s)}| \cdot \underbrace{|e^{-j\operatorname{Im} y(s)}|}_{1} = |e^{-\operatorname{Re} y(s)}|$$

max on $j\omega$ axis not $\sigma > 0$ by max modulus then

$$= e^{-\operatorname{Re} y(j\omega)}$$
$$\Rightarrow \geq \frac{1}{e^{\operatorname{Re} y(s)}} \Rightarrow \operatorname{Re} y(s) \geq 0$$

in $\sigma > 0$

$$e^{jx} = \cos x + j \sin x$$
$$|e^{jx}| = \sqrt{\cos^2 x + \sin^2 x} = 1$$

do an analytic continuation off of $s = j\omega$

$y(j\omega) \rightarrow y(s)\Big|_{\omega = s/j}$ is analytic in $\sigma > 0$

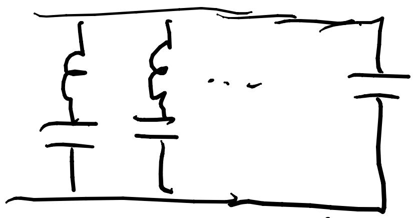
$\Rightarrow$ $y(s)$ is positive real, PR for rational case

To create a circuit, use Richards' function, p. 361

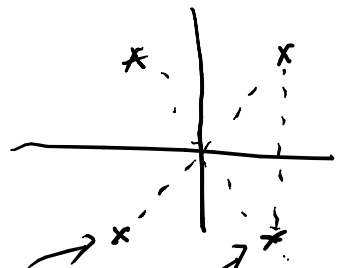
1st do LC synthesis $\Rightarrow$ lossless; $P_{in}(j\omega) \equiv 0$

$2\,Ev\,y = y(s) + y(-s) = 0$ all poles are on $s = j\omega$

$$y(s) = \sum \frac{2k_i s}{(s^2 + \omega_i^2)} + k_\infty s$$



gives this circuit
(and Foster form)



pole of y also a pole of y not allowed