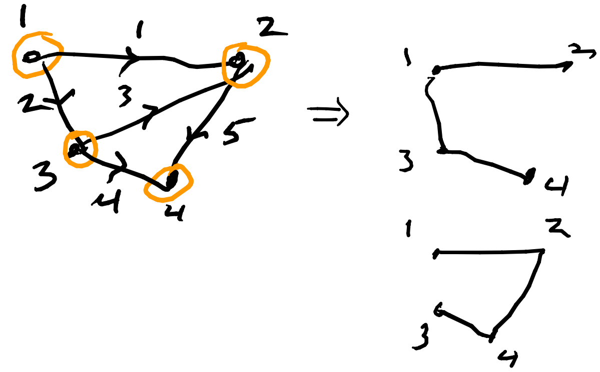
EE 610
09/17/11

Note Title 9/27/2011

incidence matrix
KCL @ nodes

$$\begin{bmatrix}0\\0\\0\\0\end{bmatrix}=\begin{bmatrix}1 & 1 & 0 & 0 & 0\\ -1 & 0 & -1 & 0 & 1\\ 0 & -1 & 1 & 1 & 0\\ 0 & 0 & 0 & -1 & -1\end{bmatrix}\begin{bmatrix}i_1\\i_2\\i_3\\i_4\\i_5\end{bmatrix}$$



$\Uparrow$
augmented
incidence matrix A_a, singular; $A_a A_a^T$ is 4×4

Semi-state

$$E\dot{x} = Ax + Bu$$
$$y = Cx$$

for the differentiator $T(s) = s = C(sE - A)^{-1}B$

$$\begin{bmatrix}0 & 0\\ 1 & 0\end{bmatrix}\dot{x} = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}x + Bu$$

$$y = Cx \quad , \quad (sE - A)^{-1} = \begin{bmatrix}-1 & 0\\ s & -1\end{bmatrix}^{-1} = \frac{1}{1}\begin{bmatrix}-1 & 0\\ -s & -1\end{bmatrix}$$

$$y = s\,u = [c_1 \; c_2]\begin{bmatrix}-1 & 0\\ -s & -1\end{bmatrix}\begin{bmatrix}b_1\\ b_2\end{bmatrix}u = [0 \; -1]\begin{bmatrix}-1 & 0\\ -s & -1\end{bmatrix}\begin{bmatrix}1\\ 0\end{bmatrix}u = s\cdot u$$

$$\begin{bmatrix}0 & 0\\ 1 & 0\end{bmatrix}\dot{x} = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}x + \begin{bmatrix}1\\ 0\end{bmatrix}u$$

$$y = [0 \; -1]x \qquad \Longrightarrow \quad T(s) = s$$

If E is nonsingular then $\dot{x} = E^{-1}Ax + E^{-1}Bu = \hat{A}x + \hat{B}u$

$$y = Cx + Du$$
$$\Rightarrow T(s) = C(s1_n - \hat{A})^{-1}\hat{B} + D$$

as $s \to \infty \Rightarrow \quad T(s) \Rightarrow \frac{C\,1_n B}{s} \Rightarrow 0$ no pole at ∞

Passive circuits $\quad P_{in} \geq 0; \quad \mathcal{E}(t) = \int_0^t p(\tau)d\tau + \mathcal{E}(0) \geq 0$ means passive

$$\mathcal{E}(t) = \int_0^t v^T(\tau)\, i(\tau)\, d\tau \geq 0 \text{ for all } t > 0$$

& all $v(t)$ & $i(t)$ supported by a circuit

When does $y(s)$ describe a passive circuit

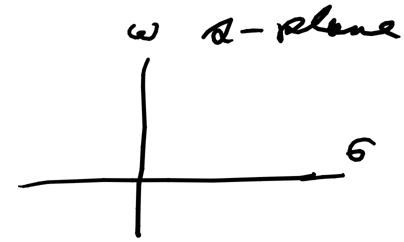
$y(s)$ needs to be positive-real

conditions are

$\operatorname{Re} y(s) \geq 0$ in $\sigma > 0$ (passive)

no singularities in $\sigma > 0$ (stable)

$y(\sigma)$ is real for $\sigma > 0$ (real elements)



$s = \sigma + j\omega$

Ex: 1) $y(s) = \sqrt{s}$ $\quad$ ∞ long passive RC line

note if $s = \sigma = -1$ then $y(-1) = \sqrt{-1} = j$ not real

but $\quad y(\sigma) = \sqrt{\sigma}$ is real if $\sigma > 0$

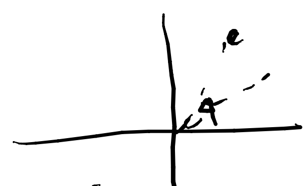
is not analytic at $s = 0$ as $\dfrac{dy}{ds} = \dfrac{1/2}{\sqrt{s}} \to \infty$ @ $s = 0$

only singularity is at $s = 0$

& $\operatorname{Re} y(s) = \dfrac{\sqrt{s} + \sqrt{s^*}}{2} = \dfrac{|s|^{1/2} e^{j\angle s/2} + |s|^{1/2} e^{-j\angle s/2}}{2}$

$= |s|^{1/2} \cos(\angle s/2) > 0$

for s in $\sigma > 0$



2) $y(s) = N(s)/D(s)$, $\quad N(s)$ & $D(s)$ polynomial

rational $y(s)$: conditions

call positive-real rational PR

Real for real s, $\sigma > 0 \Rightarrow$ real coefficients

($y(\sigma)$ real for $\sigma > 0$)

analytic in $\sigma > 0$; can only have poles for singular points $\Rightarrow$ no poles in $\sigma > 0$

but poles on $j\omega$ axis are simple & with a positive real residue

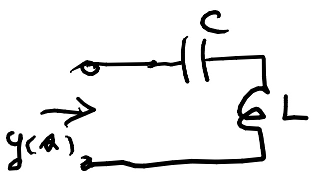
Check $\operatorname{Re} y(s)$ in $\sigma > 0 \Rightarrow \operatorname{Re} y(j\omega) \geq 0$

where we choose the boundary of $\sigma > 0$ as the $j\omega$ axis closed at ∞ on the right

Ex: 1) $y(s) = G = 1/R$

$\Rightarrow R$ real ≥ 0

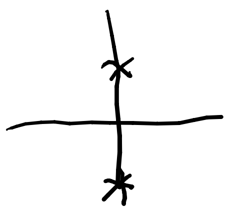
2)



$$z(s) = Ls + \frac{1}{Cs} \; ; \; y(s) = \frac{1}{z(s)} = \frac{Cs}{1 + LCs^2}$$

need LC & C real $\Rightarrow$ L & C real

poles $y(s) \Rightarrow \quad 0 = 1 + LCs^2 \Rightarrow s^2 = -1/LC \; ; \; s = \pm j\sqrt{1/LC}$



$$y(s) = \frac{s/L}{\left(s^2 + \frac{1}{LC}\right)} = \frac{(1/L)s}{\left(s + j\sqrt{1/LC}\right)\left(s - j\sqrt{1/LC}\right)}$$

$$= \frac{k_1}{s + j\sqrt{1/LC}} + \frac{k_2}{s - j\sqrt{1/LC}}$$

k_1 = residue at $s = -j\sqrt{1/LC}$ multiply by $s + j\sqrt{1/LC}$

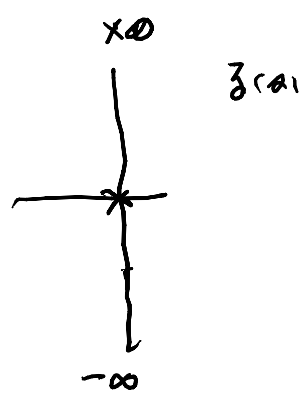
$$\left(s + j\sqrt{1/LC}\right) y(s) = \frac{1}{L} s \Big/ \left(s - j\sqrt{1/LC}\right) = k_1 + k_2 \frac{s + j\sqrt{1/LC}}{s - j\sqrt{1/LC}}$$

$$k_1 = \frac{1}{L} \frac{s}{s - j\sqrt{1/LC}} \Bigg|_{s = -j\sqrt{1/LC}} = \frac{1}{L} \frac{\left(-j\sqrt{1/LC}\right)}{-2j\sqrt{1/LC}} = \frac{1}{2L} \Rightarrow L > 0 \text{ for } k_1 \text{ positive}$$

$k_2 = 1/2L$

if $y(s)$ is PR then so is $z(s) = 1/y(s)$

here $z(s) = \frac{1}{Cs} + Ls$



$\Rightarrow C > 0$ for $y(s)$ to be PR