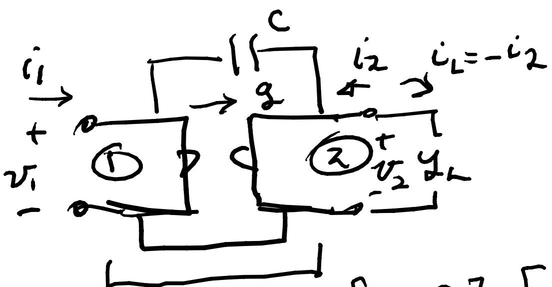
EE 610
09/22/11

Note Title 9/21/2011



$$y_{in}(y_L + sC) = sC \cdot y_L + g^2$$

$$Y_{coupling} = \begin{bmatrix} 0 & g \\ -g & 0 \end{bmatrix} + \begin{bmatrix} sC & -sC \\ -sC & sC \end{bmatrix} = \begin{bmatrix} sC & g - sC \\ -g - sC & sC \end{bmatrix}$$

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$-y_L v_2 = i_2$

$$y_{in} = \frac{sC \cdot y_L + g^2}{y_L + sC}$$

$$(-y_L - y_{22}) v_2 = y_{21} v_1$$

$$\Rightarrow v_2 = -(y_L + y_{22})^{-1} y_{21} v_1$$

$$i_1 = y_{11} v_1 + y_{12} v_2 = \left[y_{11} - y_{12} (y_L + y_{22})^{-1} y_{21} \right] v_1$$

$$\frac{i_1}{v_1} = y_{in} = y_{11} - y_{12} (y_L + y_{22})^{-1} y_{21}$$

$$= sC - (g - sC) \frac{1}{y_L + sC} \cdot (-g - sC)$$

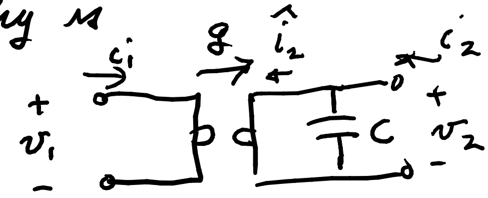
$$= \frac{sC y_L + sC^2 - (-g^2 + s^2 C^2)}{y_L + sC} = \frac{g^2 + sC y_L}{y_L + sC}$$

$$y_{in} \cdot y_L + y_{in} \cdot sC = g^2 + sC y_L$$

$$(y_{in} - sC) y_L = g^2 - y_{in} sC \Rightarrow y_L(s) = \frac{g^2 - y_{in}(s) \cdot sC}{y_{in} - sC}$$

(like a Richards' function) p.361

Why is



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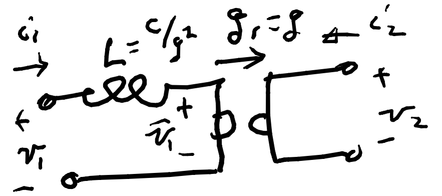


$i_1 = g v_2$
$i_2 = -g v_1$

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 0 & g \\ -g & sC \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$i_1 = g v_2$
$i_2 = sC v_2 + (-g v_1)$

$$i_1 = \frac{1}{sL}\cdot(v_1 - \hat{v}_1) = g_1 v_2 \Rightarrow \hat{v}_1 = v_1 - sLg_1 v_2$$

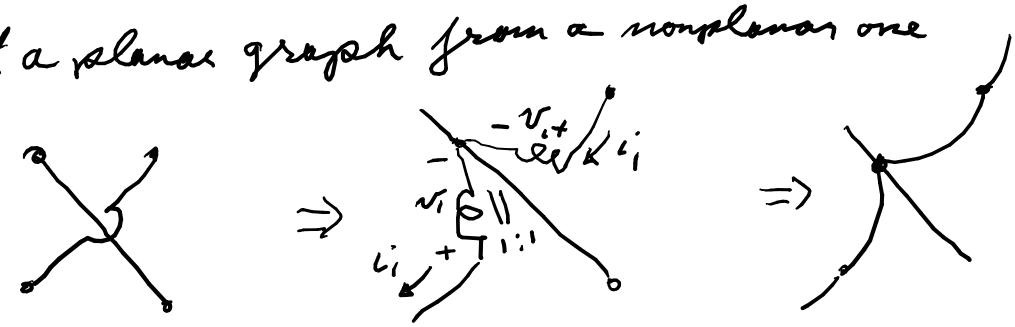
$$i_2 = -g_1 \hat{v}_1 = -g_1[v_1 - sLg_1 v_2]$$

$$\Rightarrow \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 0 & g_1 \\ -g_1 & sLg_1^2 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$$Y = \begin{bmatrix} 0 & g \\ -g & sC \end{bmatrix} = \begin{bmatrix} 0 & g_1 \\ -g_1 & sLg_1^2 \end{bmatrix} \text{ if } g_1 = g$$

$Lg_1^2 = Lg^2 = C$
$\Rightarrow L = C/g^2$

To get a planar graph from a nonplanar one



allows preservation of circuit equations

Semistate equations $E\dot{x} = Ax + Bu$
$y = Cx$

Let P = nonsingular $\Rightarrow$ $PE\dot{x} = PAx + PB\cdot u$
$y = Cx$

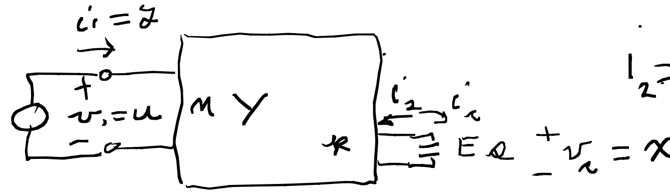
here x is a k-vector
A is $k \times k \Rightarrow P$ is $k \times k$, u is an n-vector
y is an m-vector

Let Q be nonsingular & $x = Q\hat{x}$; $\hat{x} = Q^{-1}x$

$PEQ\dot{\hat{x}} = PAQ\hat{x} + PBu$
$y = CQ\hat{x}$

preserves $T(s)U(s) = Y(s)$

$$T(s) = C(Es - A)^{-1}B = CQ(PEQs - PAQ)^{-1}PB$$
$$CQ(P[Es - A]Q)^{-1}PB$$



$I_2 = -L_x$

$i_1 = y_{11} v$

$$y = i_1 = y_{11} v_1 + y_{12} v_2 = y_{11} u + y_{12} x$$

$$i_2 = -E s v_2 = y_{21} v_1 + y_{22} v_2 = y_{21} u + y_{22} x$$

$$E\dot{x} = -y_{22} x - y_{21} u \qquad E\dot{x} = Ax + Bu$$

$$i_1 = y = y_{12} x + \underline{\underline{y_{11} u}} \qquad y = Cx + Du$$

$$\Downarrow$$

$$Y = \begin{bmatrix} D & C \\ -B & -A \end{bmatrix}$$

if $E = 1_k$

$$Y = \underbrace{\frac{(Y+Y^T)}{2}}_{\substack{\text{symmetric} \\ \text{uals} \\ \text{resistors}}} + \underbrace{\frac{(Y-Y^T)}{2}}_{\substack{\text{skew symmetric} \\ \text{gyrators}}}$$

given $\dot{x} = Ax + Bu$, $y = Cx + Du$ $\Rightarrow$ $E\dot{\hat{x}} = \hat{A}\hat{x} + \hat{B}u$, $y = \hat{C}\hat{x}$