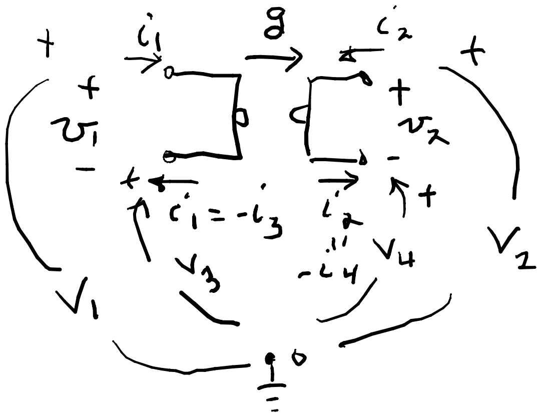
Note Title 9/20/2011

EE610
09/20/11
corrections 09/21/11

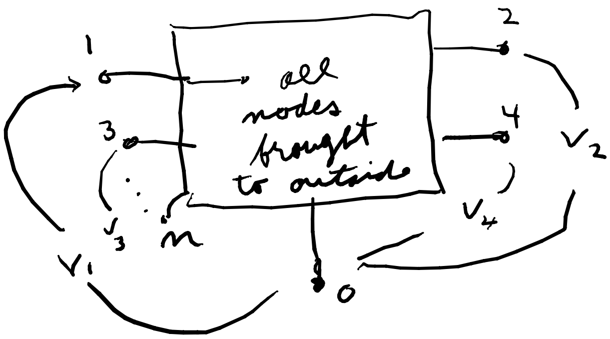


$$Y = \begin{bmatrix} 0 & g \\ -g & 0 \end{bmatrix} = -Y^T$$

$v_1 = V_1 - V_3$, $v_2 = V_2 - V_4$

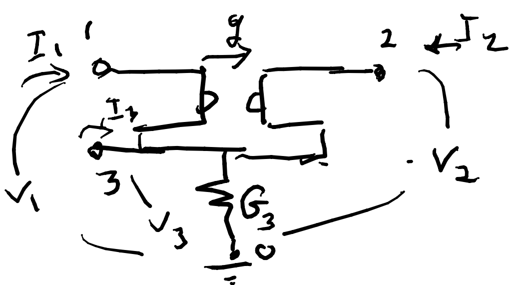
$I_1 = i_1$ $\quad I_2 = i_2$

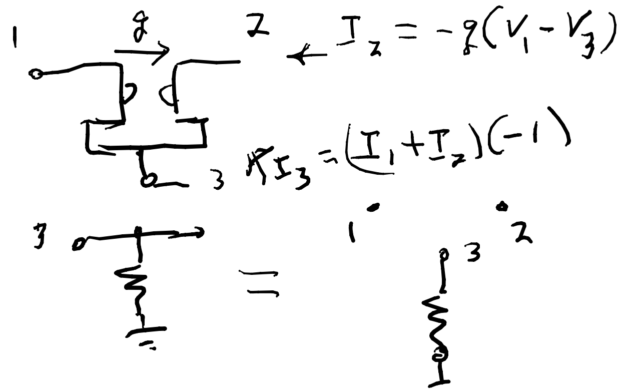
$I_3 = -i_1$ $\quad I_4 = -i_2$



$$I = Y_{nodal} V$$

Ex:



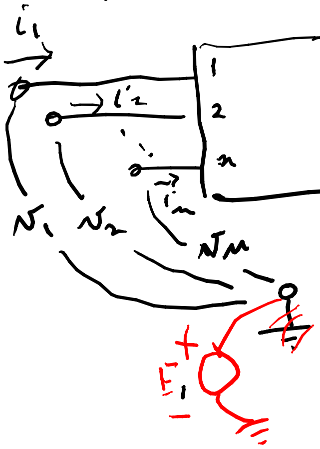


$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} \quad Y_{nodal} \text{ is } 3\times 3 = \underbrace{\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & G_3 \end{bmatrix}}_{\text{for } G_3} + \underbrace{\begin{bmatrix} 0 & g & -g \\ -g & 0 & g \\ g & -g & 0 \end{bmatrix}}_{\text{for the gyrator}} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

(indefinite; Σ of row entries = 0
Σ of column entries = 0)

$$Y_{nodal} = \begin{bmatrix} 0 & g & -g \\ -g & 0 & g \\ g & -g & G_3 \end{bmatrix}$$

to get in general use indefinite admittances move ground off of circuit



by KCL $\sum_{j=1}^{m} i_j = 0$; $i = Y_{ind}\, v$

$$= \begin{bmatrix} i_1 \\ i_2 \\ \vdots \\ i_m \end{bmatrix} = \begin{bmatrix} y_{11}v_1 + y_{12}v_2 \cdots \\ y_{21}v_1 + y_{22}v_2 \cdots \\ \vdots \\ y_{m1}v_1 \quad \cdots \end{bmatrix}$$

shows if choose $v_k = 1$, $k \neq j$, $v_j = 0$

$\sum_{j=1}^{m} y_{jk} \cdot 1 = 0$ for a fixed k for any k

$1 \le k \le m$

$\Rightarrow$ sum of entries in any column of this $Y = 0$

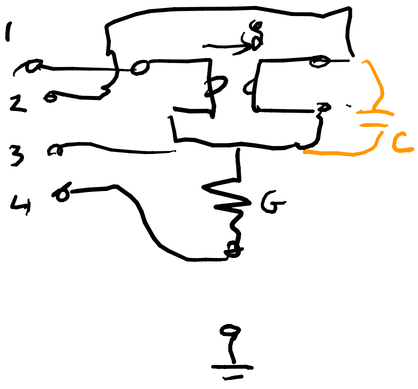
$i = Yv + YE$ choose the $v = 0$, $E = \begin{bmatrix} E_1 \\ \vdots \\ E_1 \end{bmatrix}$

$\Rightarrow$ sum of $y_{11}E_1 + y_{12}E_1 + \cdots y_{1m}E_1 = 0$

or Σ of y's in a row $= 0$

for this Y, call the indefinite Y_g, the sum of entries in a row is 0 and in a column is zero

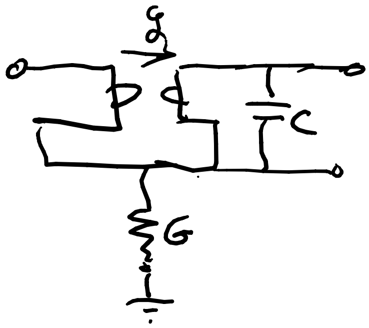
Ex:



$$Y_{ind} = \begin{bmatrix} 0 & g & -g & 0 \\ -g & 0 & g & 0 \\ g & -g & G & -G \\ 0 & 0 & -G & G \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & sC & -sC & 0 \\ 0 & -sC & sC & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

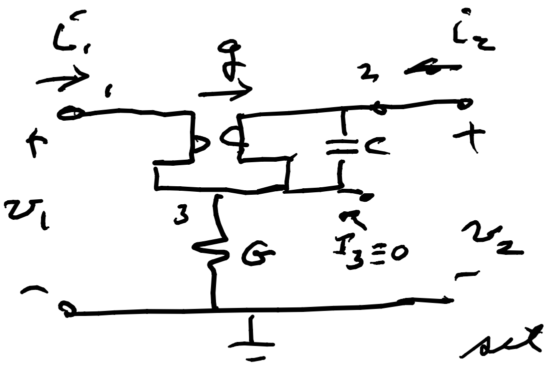
If bring the ground to node k, sets $v_k = 0$ so can remove the kth column (as multiply by 0) can remove the kth row as the $i_k = \sum_{j \neq k} i_j$

gives Y_{nodal}. If want port descriptions we need to "remove" internal nodes



$$Y_{nodal} = \left[\begin{array}{cc|c} 0 & g & -g \\ -g & sC & g-sC \\ \hline g & -g-sC & G+sC \end{array}\right]$$

grounded node 4



$$\begin{bmatrix} I_1 \\ I_2 \\ \hline I_3 \end{bmatrix} = \left[\begin{array}{c|c} Y_{11} & Y_{12} \\ \hline Y_{21} & Y_{22} \end{array}\right] \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

set $I_3 = 0$

$$I_3 = 0 = Y_{21} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} + Y_{22} \cdot V_3$$

$$\Rightarrow V_3 = -Y_{22}^{-1} Y_{21} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

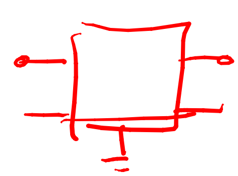
$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = Y_{11} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} + Y_{12} \cdot V_3 = \left[Y_{11} - Y_{12} Y_{22}^{-1} Y_{21} \right] \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$$\Rightarrow Y_{2\text{-port}\ (2\times 2)} = Y_{11} - Y_{12} Y_{22}^{-1} Y_{21}$$

$$= \begin{bmatrix} 0 & g \\ -g & sC \end{bmatrix} - \begin{bmatrix} -g \\ g-sC \end{bmatrix} \frac{1}{G+sC} \begin{bmatrix} g & -g-sC \end{bmatrix}$$

$$Y_{2\text{-port}} = \begin{bmatrix} 0 & g \\ -g & sC \end{bmatrix} + \underbrace{\begin{bmatrix} -g \\ g-sC \end{bmatrix} \begin{bmatrix} -g & g+sC \end{bmatrix}}_{\begin{bmatrix} g^2 & -g^2 - g sC \\ -g^2 + sCg & g^2 - s^2C^2 \end{bmatrix} \frac{1}{G+sC}} \frac{1}{G+sC}$$

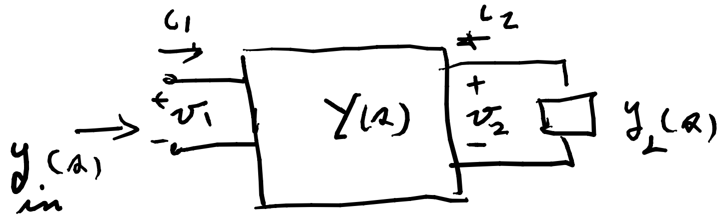
$$Y(s) = \begin{bmatrix} \dfrac{g^2}{G+sC} & g + \dfrac{-g^2 - g sC}{G+sC} \\ -g - \dfrac{g^2 - g sC}{G+sC} & sC + \dfrac{g^2 - s^2C^2}{G+sC} \end{bmatrix}$$



$$= \frac{1}{s+G/C} \begin{bmatrix} g^2/C & -\frac{g}{C}(g-G) \\ -\frac{g}{C}(g+G) & \frac{g^2}{C} + sG \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & G \end{bmatrix}}_{\text{residue at } \infty} + \frac{1}{s+G/C} \begin{bmatrix} g^2/C & -\frac{g}{C}(g-G) \\ -\frac{g}{C}(g+G) & \frac{g^2-G^2}{C} \end{bmatrix}$$

singular residue @ $s = -G/C \Rightarrow \delta[Y] = 1$

assume we got a 2-port $Y(s)$



find y_{in} given $Y(s)$ & $y_L(s)$

$$-i_2 = y_L(s) v_2 = -y_{21} v_1 - y_{22} v_2$$

solve for v_2 in terms of $v_1 \Rightarrow \quad -y_{21} v_1 = (y_L + y_{22}) v_2$

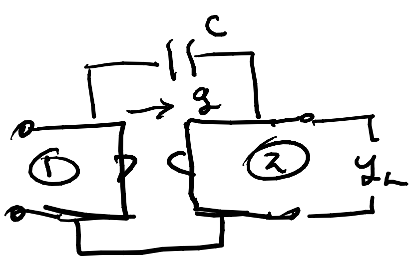
$$v_2 = \frac{-y_{21}}{y_L + y_{22}} v_1 \quad ; \quad i_1 = y_{11} v_1 + y_{12} v_2$$

$$= (y_{11} - y_{12}(y_L + y_{22})^{-1} y_{21}) v_1$$

$$\frac{i_1}{v_1} = \frac{y_{11} y_L + y_{11} y_{22} - y_{12} y_{21}}{y_L + y_{22}} = \frac{y_{11} y_L + \Delta_y}{y_L + y_{22}}, \quad \Delta_y = \det Y = y_{11} y_{22} - y_{12} y_{21}$$

$$y_{in} = \frac{y_{11} y_L + \Delta_y}{y_L + y_{22}}$$

Ex:



$$\Rightarrow \quad Y_{2\text{-port}} = \begin{bmatrix} sC & g - sC \\ -g - sC & sC \end{bmatrix}$$

$$\Delta_y = (sC)(sC) - (g - sC)(-g - sC)$$

$$= (sC)^2 - (-g^2 + s^2 C^2)$$

$$= g^2$$

$$y_{in} = \frac{sC \cdot y_L + g^2}{y_L + sC}$$