EE 610
09/13/11

Note Title 9/11/2011

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & C & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \frac{dx}{dt} = \begin{bmatrix} -G & 0 & +1 & -1 \\ 0 & 0 & 0 & +1 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & +1 & 0 \end{bmatrix} x + \begin{bmatrix} G \\ 0 \\ 0 \\ 0 \end{bmatrix} e_1 \Rightarrow \underbrace{\mathcal{E}\frac{dx}{dt} = \mathcal{A}x + \mathcal{B}u}_{\text{semi-state equations}}$$

$$\underset{\text{output}}{\overset{\shortparallel}{y}} = \mathcal{C}x \quad , \quad x = \begin{bmatrix} v_t \\ i_\ell \end{bmatrix} \doteq \text{semistate}$$

$u = e_1$ (a scalar here)

$$v_4 = \begin{bmatrix} +1 & -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ i_3 \\ i_4 \end{bmatrix}_b$$

4th Eqn: $i_3 \equiv 0 \Rightarrow di_3/dt = 0$

3rd: $-v_1 = 0 \Rightarrow \frac{dv_1}{dt} = 0, \Rightarrow v_4 = -v_2$

1st: $0 = -Gv_1 + i_3 - i_4 + Ge_1 \Rightarrow \underline{-Gv_1 + Ge_1 - i_4 = 0}$

2nd: $Csv_2 = +i_4 = +Ge_1$

$$\text{output} = v_4 = -v_2 = -\frac{Ge_1}{sC} \Rightarrow \frac{\text{output}}{\text{input}} = \frac{v_4}{e_1} = -\frac{G}{sC} = T(s)$$

= transfer function

Can normally get semi-state equations

$$E\frac{dx}{dt} = Ax + B\overset{\text{input}}{\overset{\shortparallel}{u}} \qquad \Rightarrow \qquad E\frac{dx}{dt} = \mathcal{A}(x,t) + Bu$$

$$\underset{\text{output}}{\overset{\shortparallel}{y}} = C\underset{\text{semistate}}{x} \qquad\qquad y = Cx$$

generalizes
(nonlinear, time-varying)

Linear time invariant

$$T(s)\cdot V = Y$$

$$E\, s X(s) = A X(s) + B U(s), \qquad X(s) = \int_{-\infty}^{\infty} x(t) e^{-st}\, dt$$

$$Y = C X(s) \qquad\qquad = \mathcal{L}[x(t)]$$

$$(Es - A) X(s) = B U(s)$$

$$X(s) = (sE - A)^{-1} B \cdot U(s)$$

$$Y(s) = C(sE - A)^{-1} B \cdot U(s) \Rightarrow T(s) = C(sE-A)^{-1}B$$

$$C = [1,-1,0,0], \quad E = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & C & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad A = \begin{bmatrix} -G & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} G \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$T(s) = [1 \;\; -1 \;\; 0 \;\; 0] \begin{bmatrix} G & 0 & -1 & 1 \\ 0 & sC & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix}^{-1} \begin{bmatrix} G \\ 0 \\ 0 \\ 0 \end{bmatrix}; \qquad \begin{aligned} (1,1) &= \Delta_{11}/\Delta \\ (2,1) &= \Delta_{12}/\Delta \end{aligned}$$

$$\Delta = G[sC]\begin{vmatrix} 0 & 0 \\ -1 & 0 \end{vmatrix} + 1\cdot[sC]\begin{vmatrix} -1 & 1 \\ -1 & 0 \end{vmatrix} = 0 + sC\, x(+1) = sC$$

$$\Delta_{11} = 0, \quad \Delta_{12} = \begin{vmatrix} 0 & 0 & -1 \\ 1 & 0 & 0 \\ 0 & -1 & 0 \end{vmatrix} = 1\times(-1)^{2+1}\cdot\begin{vmatrix} 0 & -1 \\ -1 & 0 \end{vmatrix} = -1\times(-1) = +1$$

$$T(s) = [1 \;\; -] \begin{bmatrix} 0 \\ 1/sC \end{bmatrix} G = -\frac{G}{sC}$$

If every component has an admittance

$$\underset{\substack{\| \\ Y(s)_{b\times b}}}{A(s)}\, V(s) = \underset{\substack{\| \\ \mathbb{1}_b}}{B(s)}\, I(s) \qquad \mathbb{1}_b = b\times b \text{ identity} = \begin{bmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{bmatrix}$$

$$Y_{b\times b}\, V(s) = I(s)$$

$$V_b = V + E \leftarrow \text{voltage source}$$

$$V_b = C^T V_t, \quad 0_t = C I_b \qquad I_b = I + J \leftarrow \text{current source}$$

$$Y_{b\times b}(V_b - E) = I_b - J$$

$$\mathcal{C}\, Y_{b\times b}(\mathcal{C}^T V_t - E) = \mathcal{C} I_b - \mathcal{C} J = -\mathcal{C} J$$

$b = t + l$

$$\underbrace{\mathcal{C}\, Y_{b\times b}\, \mathcal{C}^T}_{\text{admittance}} \cdot V_t = \underbrace{\mathcal{C}\, Y_{b\times b} E - \mathcal{C} J}_{\text{Norton equivalents}}$$

Thevenin

(nodal admittance if tree branches all to ground)