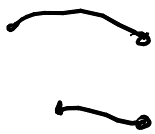
EE 610
09/08/11
correction 09/11/11

Note Title 9/8/2011

graph = set of lines & dots (= nodes)
(lines have dots at each end)

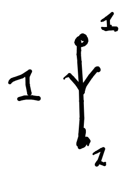
connected graph = travel on lines from any node to any other



not connected

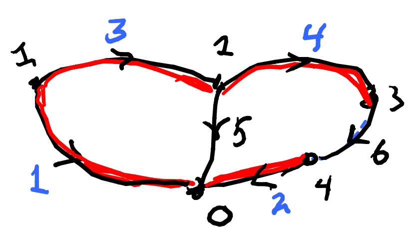
b = # of lines = branches, n = # dots = nodes

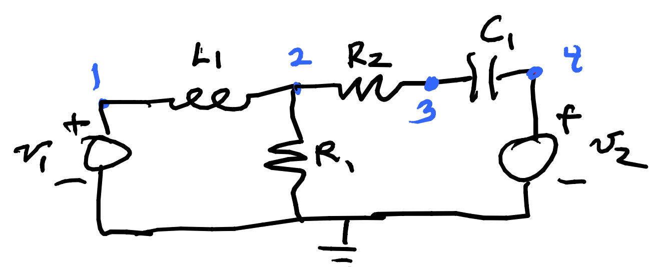
oriented graph, put an arrow on each branch



numbered, put a unique number on each branch and on each node

a tree in a connected graph = set of branches on which can travel to any node but no closed paths



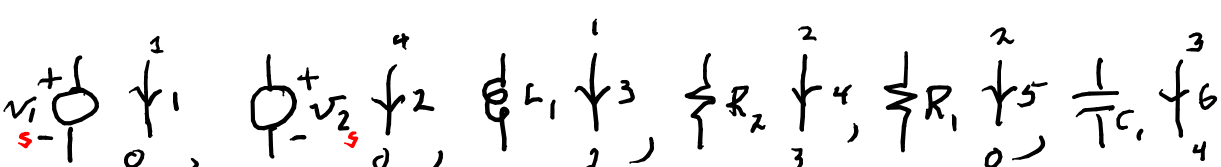


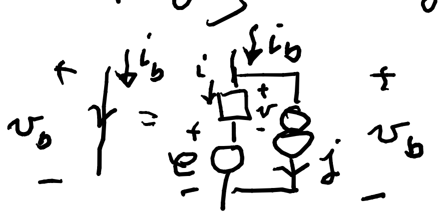
→ tree branch

$t=4$, $n=5$, $b=6$, $s=1$ = # of separate parts

$n=t+1$, $b=t+\ell$, $\ell=2$

$0_4 = C i_b$, $0_2 = \mathcal{J} v_b$, $v_b = C^T v_t$, $i_b = \mathcal{J}^T i_\ell$ } laws of connection





$i_b = i + j$

$v_b = v + e$

e = voltage sources

j = current sources

$$i_b = \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ i_6 \end{bmatrix}_b \qquad e = \begin{bmatrix} v_{1_s} \\ v_{2_s} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} ; \quad j = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} ;$$

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{L_1 s} & 0 & 0 & 0 \\ 0 & 0 & 0 & G_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & G_1 & 0 \\ 0 & 0 & 0 & 0 & 0 & sC_1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ i_6 \end{bmatrix}}_{\text{linear components}}$$

branch by branch admittance sub matrix

linear components
$A v = B i$

$$v = v_b - e \ , \quad i = i_b - j \ , \quad A v = B i$$

$$\Downarrow$$

$$A v_b - A e = B i_b - B j$$

$$A v_b - B i_b = A e - B j$$

$$A \mathcal{C}^T v_t - B \mathcal{J}^T i_\ell = A e - B j$$

$$= \underbrace{\left[A\mathcal{C}^T , -B\mathcal{J}^T \right]}_{\text{circuits}} \underbrace{\begin{bmatrix} v_t \\ i_\ell \end{bmatrix}}_{\text{unknowns}} = \underbrace{Ae - Bj}_{\text{sources}} \quad \Leftarrow \text{circuit equations for linear circuits}$$

$$= \left[\mathcal{E}(s) - \mathcal{A} \right] x = u$$

u = input
x = semi state, if $s = d/dt$

old example:

$$e = \begin{bmatrix} e_1 \\ 0 \\ 0 \\ 0 \end{bmatrix} , \quad i_b = \mathcal{J}^T i_\ell = \begin{bmatrix} -1 & -1 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} i_\ell \ , \quad v_b = \mathcal{C}^T v_t = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 1 & -1 \end{bmatrix} v_t$$

$$\begin{bmatrix} G & 0 & 0 & 0 \\ 0 & sC & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix}$$

$$\begin{bmatrix} G & 0 & 0 & 0 \\ 0 & sC & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 1 & -1 \end{bmatrix} v_t - \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} i_\ell =$$

$$\begin{bmatrix} G & 0 & 0 & 0 \\ 0 & sC & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} e_1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} Ge_1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

correction

$$\left[\begin{array}{cc|cc} G & 0 & -1 & +1 \\ 0 & sC & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{array}\right] \begin{bmatrix} v_t \\ \hline i_\ell \end{bmatrix} = \begin{bmatrix} Ge_1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \left\{ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & sC & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} G & 0 & -1 & +1 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix} \right\} x$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & C & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \frac{dx}{dt} = \begin{bmatrix} -G & 0 & +1 & -1 \\ 0 & 0 & 0 & +1 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & +1 & 0 \end{bmatrix} x + \begin{bmatrix} G \\ 0 \\ 0 \\ 0 \end{bmatrix} e_1 \Rightarrow \underbrace{\mathcal{E} \frac{dx}{dt} = \mathcal{A}x + \mathcal{B}\underset{\text{input}}{u}}_{\text{semi-state equations}}$$

$$\underset{\text{output}}{y} = \mathcal{C}x, \quad x = \begin{bmatrix} v_t \\ i_\ell \end{bmatrix} \triangleq \text{semistate}$$

$u = e_1$ (a scalar here)